The Fishnet Sequence.

OEIS A359369, a sequence of David Sycamore · Megavissey, England.

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Abstract.

We examine a simple sequence that exhibits trajectories in scatterplot associated with binary weight, implying that a given value m appears at most $\tau(m)$ times in the sequence.

Introduction.

Consider OEIS A120(x), the binary weight of x, that is, the sum of the binary digits or bits of x_2, the binary expansion of x. We consider A359369, a sequence of Sycamore defined as follows:

$$a(1) = 1. \text{ For } n > 1,\ a(n) = \sum_{k=1}^{n} \text{A120}(a(k))$$
$$\text{such that A120}(a(k)) = \text{A120}(a(n)).$$

We can rewrite this definition in logic thus:

$$\text{Let } w = \text{A120}(n).$$
$$\text{Define function } f(n) = w \times c(w),$$
$$\text{where } c(w) = |\, k \in a(1 \ldots n) \,| : \text{A120}(k) = w.$$
$$a(1) = 1,\ a(n) = f(a(n-1)).$$

The sequence begins as follows:

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1, 1, 2, 3, 2, 4, 5, 4, 6, 6, 8, 7, 3, 10, 12, 14, 6,
16, 8, 9, 18, 20, 22, 9, 24, 26, 12, 28, 15, 4, 10, 30,
8, 11, 18, 32, 12, 34, 36, 38, 21, 24, 40, 42, 27, 12,
44, 30, 16, 13, 33, 46, 20, 48, 50, 36, 52, 39, 24, 54,
28, 42, 45, 32, 14, 48, 56, 51, 36, 58, ...
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This sequence has a curious plot that resembles a fishnet. It is clear that $a(n)$ is odd iff both w and $c(w)$ odd.

Lemma 1. Let trajectory $T_w \ni a(j)$: A120$(a(j)) = w$. Suppose A120$(a(i))$ = A120$(a(j)) = w$, $i < j$, and that there is no $a(k)$, $i < k < j$, such that A120$(a(k)) = w$. Then $c(w, j) = c(w, i) + 1$.

Proof. Suppose not. Suppose that $c(w, j) \le c(w, i)$. This would suggest that A120$(a(j)) = w$ did not increase the number of terms in $a(1 \ldots j)$ that have binary weight w, contradicting the definition of $c(w)$. Now suppose that $c(w, j) > c(w, i) + 1$. This implies that there is at least one term $a(k)$, $i < k < j$, such that A120$(a(k)) = w$, contradicting the proposition. In fact, if $a(j)$ represents the next number in the sequence after $a(i)$ with the same binary weight w, then the proposition must be true. ■

Corollary 1.1. Let $T_w(x)$ be the x-th term in a that has binary weight w. The mapping $c : T_w(x) \to y$ increments as x increments, and is tantamount to the sequence of natural numbers A27. So long as we have numbers with binary weight w, the trajectory T_w represents a set that is countably infinite.

Define sequence $S_w = \{\, m$: A120$(m) = w\}$. Therefore, $S_0 = \{0\}$, S_1 = A79, S_2 = A018900, S_3 = A014311, etc.

Corollary 1.2. The sequence consists of trajectories as follows:

$$T_w \ni a(j) : a(j) \in S_w.$$

Corollary 1.3. Since input for the sequence function is the previous output, the sequence runs through $w \in \mathbb{N}$, therefore the sequence is infinite and well-defined.

It is evident the product m appears in the sequence no more than $\tau(m)$ times. From observation, the last time $m > 1$ appears in this sequence is immediately following a number with binary weight m.

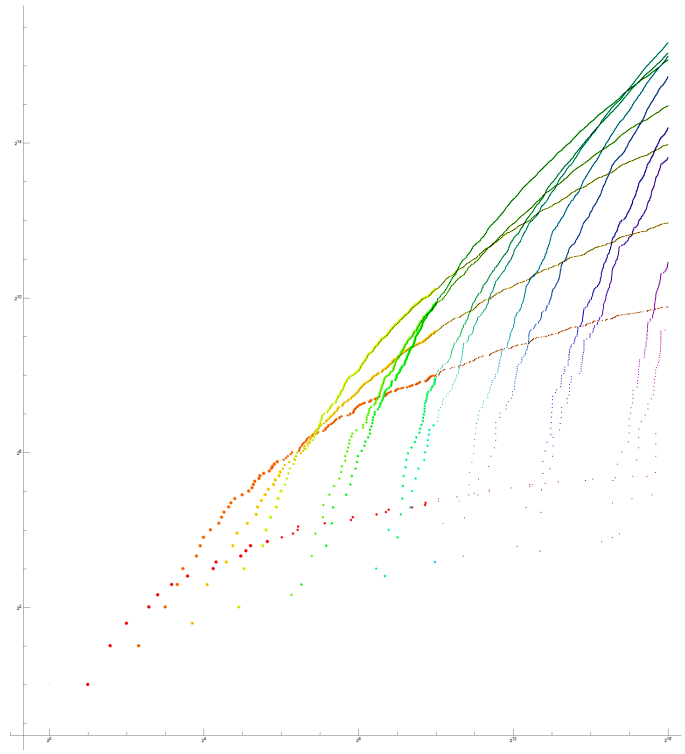

Figure 1: Log log scatterplot of $a(n)$ for $n = 1 \ldots 2^{16}$, with a color function showing w = A120$(a(n-1))$ with $w = 1$ in red, $w = 2$ in orange, …, $w = 15$ in magenta.

Conclusion.

The sequence A359369 is a simple study of the product of the "payload function" A120$(a(n-1)) = m$ and the cardinality of m in $a(1 \ldots n-1)$. Multiplication introduces the maximum cardinality $\tau(m)$ and parity patterns in the dataset. Open questions relate to the pairings of cardinality $c(w)$ and weight w, which was studied in part in December 2022 without conclusion.

References:

[1] N. J. A. Sloane, *The Online Encyclopedia of Integer Sequences*, retrieved December 2022.

Code:

[C1] Generate A359369:

```
nn = 2^10; c[_] = 0; a[1] = 1;
f[n_] := DigitCount[n, 2, 1];
Do[Set[k, ( c[#]++; # c[#]) &[f[#]]] &@ a[n - 1];
  Set[a[n], k], {n, 2, nn}];
Array[a, nn]
```

Concerns sequences:

A000005: Divisor counting function $\tau(n)$.
A000079: Powers of 2.
A000120: Binary or Hamming weight WT(n).
A014311: Sums of 3 distinct powers of 2.
A018900: Sums of 2 distinct powers of 2.

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