# On a coprimality conditional LES yielding multiples of primes.

**A sequence of David Sycamore · Windhoek, Namibia.**

**Michael Thomas De Vlieger · St. Louis, Missouri · 11 February 2023.**

## Abstract.

We examine a coprimality conditional lexically earliest sequence (LES) that uses three previous terms to determine output. Several aspects are evident consequences of definition. We use constitutive states to attempt to determine origins of certain features evident in scatterplot. This cursory analysis relies upon concepts laid out in [2].

## Introduction.

David Sycamore originally defined the sequence S20230211 thus: $a(n) = n$ for $n \le 3$; let $h = a(n-3)$, $i = a(n-2)$, and $j = a(n-1)$. For $n > 3$: $(h, j) = 1$ implies $a(n)$ is the least novel multiple of the greatest prime divisor of $i$. Else, $(h, j) \ne 1$ implies $a(n)$ is the least multiple of the smallest prime which divides neither $h$ nor $j$.

We logically define sequence S20230211 as follows:

$a(n) = n$ for $n \le 3$;
Let $h = a(n-3)$, $i = a(n-2)$, and $j = a(n-1)$.
Define function $c(x)$ to be TRUE iff $a(j) = x$ for $j < n$, else FALSE.
For $n > 3$, we define $a(n)$ according to the following axioms:
Axiom 0 [A0]: $h \perp j$ implies $a(n) =$ ⧄$k : mp \land \neg c(k)$, $p = \mathrm{GPF}(i)$.
Axiom 1: $h \sqcup j$ implies $a(n) =$ ⧄$k : mq \land \neg c(k)$, ⧄$q : q \perp hj$.

The sequence begins as follows:

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1, 2, 3, 4, 6, 5, 9, 10, 12, 15, 7, 20, 14, 18, 21, 25,
28, 30, 35, 24, 42, 11, 49, 40, 56, 27, 63, 45, 8, 50,
70, 33, 77, 36, 55, 16, 60, 84, 65, 91, 75, 22, 80, 98,
39, 105, 44, 112, 66, 85, 88, 90, 119, 95, 126, 100, 48,
110, 51, 115, 54, 120, 133, 125, 140, 57, 147, 99, 26,
121, 32, 69, 34, 72, 130, 78, 135, 154, 145, 38, 81,
19, 87, 46, 29, 52, 93, 13, 96, 150, 102, 155, 161, 31,
58, 62, 108, 160, 111, 165, 168, 170, 175, 132, 182,
114, 180, 143, 185, 189, 37, 64, 74, 117, 148, 123, ...
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The following explains how the first dozen terms arise.

$a(4) = 4$ since $h = 1$ is coprime to $j = 3$, hence [A0] requires a multiple of $p = \mathrm{GPF}(i) = 2$; since 2 is new to the sequence, $m = 1$, and we have $k = 1 \times 2$.

$a(5) = 6$ since $2 \sqcup 4$ has [A1], which requires the smallest prime q that does not divide $2 \times 4 = 8$, which is 3; since $a(3) = 3$, we have $m = 2$, therefore $mq = 2 \times 3 = 6$.

$a(6) = 5$ since $3 \sqcup 6 \rightarrow$ [A1] $\rightarrow 5 \perp 3 \times 6 \land m = 1 \rightarrow mq = 1 \times 5 = 5$.

$a(7) = 9$ since $4 \perp 5 \rightarrow$ [A0] $\rightarrow \mathrm{GPF}(6) = p = 3$; 3 and 6 have already appeared, hence $m = 3$, and $mp = 3 \times 3 = 9$.

$a(8) = 10$ since $6 \sqcup 9 \rightarrow$ [A1] $\rightarrow 5 \perp 54 \land m = 2 \rightarrow mq = 2 \times 5 = 10$.

$a(9) = 12$ since $5 \sqcup 10 \rightarrow$ [A1] $\rightarrow 3 \perp 50 \land m = 4 \rightarrow mq = 4 \times 3 = 12$.

$a(10) = 15$ since $9 \sqcup 12 \rightarrow$ [A1] $\rightarrow 5 \perp 108 \land m = 5 \rightarrow mq = 5 \times 3 = 15$.

$a(11) = 7$ since $10 \sqcup 15 \rightarrow$ [A1] $\rightarrow 7 \perp 150 \land m = 1 \rightarrow mq = 1 \times 7 = 7$.

$a(12) = 20$ since $12 \perp 7 \rightarrow$ [A0] $\rightarrow \mathrm{GPF}(15) = p = 5$; $m = 4 \rightarrow mp = 4 \times 5 = 20$, etc.

## Consequences of Definition.

The axiomatic function $f(x) \rightarrow y$ is conditional upon coprimality between $h$ and $j$. Output is either dependent on $\mathrm{GPF}(i)$ in the affirmative, else ⧄$q : q \nmid hj$. In either case we place ⧄$m \times$ PRIME.

The following are evident:

$m = 1$ implies prime $k$, else composite $k$.
[A0] implies $i \sqcup k$ and with lexical axiom, $i \ne k$.
[A1] does not restrict $hj$ versus $k$.

As regards both $hj$ versus $k$ and $i$ versus $k$, coprimality outside of given terms derives from [A1].

## Constitutive Analysis.

From observation, given $2^{24}$ terms, we see the following relation between $hj$ versus $k$:

Table A.

| State | Card. | n | h | j | rad(hj) | k |
|---|---|---|---|---|---|---|
| (0) | 8447583 | 4 | 1 | 3 | 3 | 4 |
| (1) | 8329558 | 8 | 6 | 9 | 6 | 10 |
| (2) | 32 | 487 | 686 | 705 | 9870 | 47 |
| (3) | 36 | 271 | 309 | 166 | 51294 | 332 |
| (7) | 4 | 5 | 2 | 4 | 2 | 6 |

Divisor-first states are repressed in [A0] due to $k = mp$, since, unless $hj = 1 \times 2$ and given lexical axiom, the product is always composite, while $m \ge 1$ and we have relatively small multiples of $p = \mathrm{GPF}(i)$. Furthermore, divisor-first states do not result from [A1] by definition; $hj \sqcup k$ implies $hj \lozenge k$. Therefore $hj$ never divides $k$.

Through inspection, we have 2 ⑦ 6 = $a(5)$ via [A1]. We never see states ⑧ or ⑨ likely on account of the “scale issue” (related to the *abc* conjecture).

State ① results from either axiom but is dominated by [A1]. State ② arises from [A0], often involving primes. State ③ arises from [A0].

Axiom 0 implies $i \sqcup k$, but the relationship is largely ungoverned:

Table B.

| State | Card. | n | i | k |
|---|---|---|---|---|
| 0 | 4612675 | 1 | 1 | 2 |
| 1 | 11598757 | 10 | 10 | 15 |
| 2 | 341244 | 82 | 38 | 19 |
| 3 | 120384 | 7 | 6 | 9 |
| 4 | 66867 | 5 | 3 | 6 |
| 6 | 22 | 4 | 2 | 4 |
| 7 | 37262 | 9 | 9 | 12 |
| 9 | 2 | 678 | 608 | 722 |

From observation, $i \lozenge k$ is favored; $i \perp k$ arises from [A1]; it is forbidden by [A0]. State ⑤ is forbidden by lexical axiom. State ⑧ never arises in the dataset. The state, $i$ ¦| $k$, is likely closed due to scale issues. It could have arisen given $(i, k) = \{(4, 2), (9, 3), (12, 6)\}$, etc., but structurally those did not happen. Let $\mathrm{RAD}(n) =$ A7947$(m)$. Circumstance would require [A0] to have $i = p^{\varepsilon}$ and $m = 1$, or $i = s\varkappa$, $\varkappa$ squarefree and $\mathrm{RAD}(s) \mid \varkappa$, and either $\mathrm{RAD}(m) = \varkappa$ or $\mathrm{RAD}(m) = \varkappa/p$, and is probably limited by the scale issue. There might be an [A1]-source of state ⑧.

State ① arises from either axiom, often involving primes, but [A1] dominates.

State ② appears to be the fruit of [A0].

State ③ primarily results from [A0].

State ④ results from [A1] for $n \in \{5, 9, 13\}$, otherwise from [A0].

State ⑥ results from [A0].

State ⑦ results from either axiom, more often [A1] for $n \le 120$, but overall, predominantly from [A0].

State ⑨ results from [A0] to yield $a(678)$ and $a(575297)$.

$\mathrm{PRIME}(t) = k > 2$ arises from [A1] for $t < 6$. Axiom 1 requires $\mathrm{RAD}(hj) = P(t-1) =$ A2110$(t-1)$, and since $hj \ge \mathrm{RAD}(hj)$, it should yield a prime increasingly late compared to the [A0] origin associated

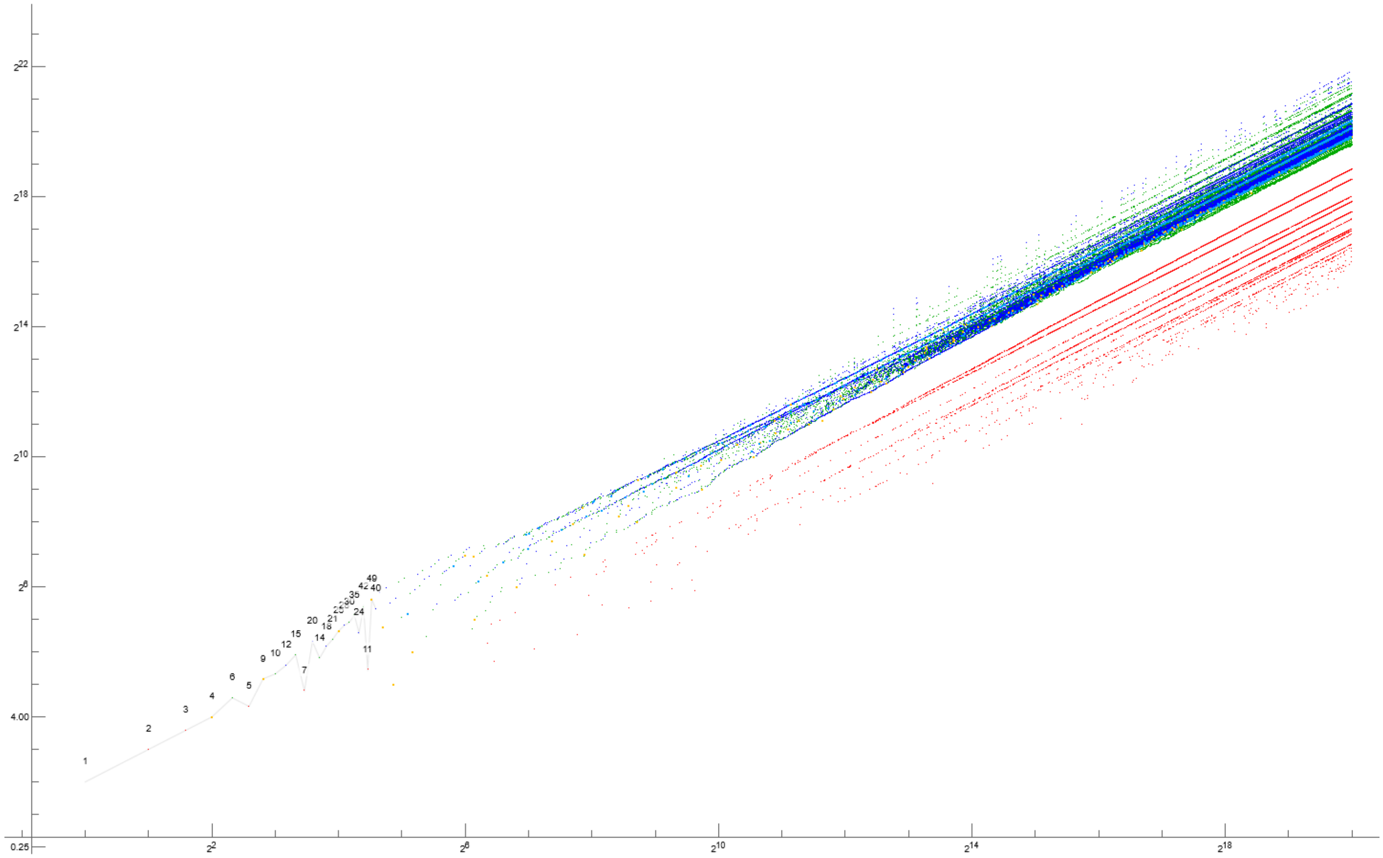


*Figure 1: Log log scatterplot of* s20230211 *for* $n = 1 \ldots 2^{20}$*, showing primes in red, multus (composite prime powers in* A246547*) in gold, varius (squarefree composites in* A120944*) in green, tantus (neither squarefree nor prime power in* A126706*) in blue, and plenus (products of multus in* A286708*, a subset of tantus) in light blue. Powerful numbers (*A1694*) are the union of {1}, the multus, and the plenus numbers. Essentially, zone* β *appears in red, with everything else in zone* α*.*

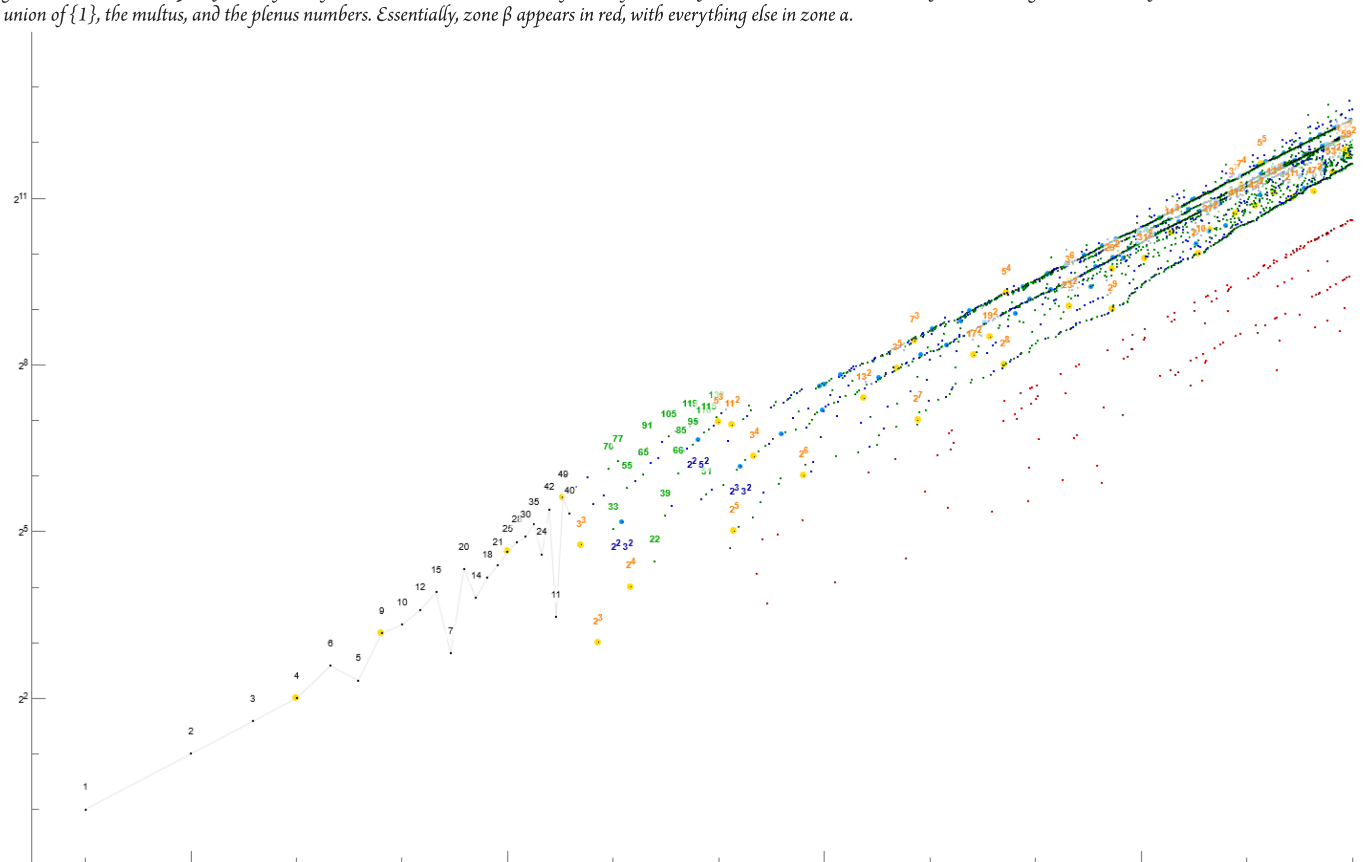


*Figure 2: Log log scatterplot of* s20230211 *for* $n = 1 \ldots 2^{12}$*, using the same color function as above, annotated.*

with $p = \text{GPF}(i)$. Therefore, though these axioms admit prime output, nearly all primes result from [A0], that is, $p = \text{GPF}(i)$.

The first indices of $\text{RAD}(hj) \in$ A2110$(1 \dots 7)$ appear below:

```
5, 6, 11, 22, 428, 256, 2608
```

## Features of Scatterplot.

Scatterplot arranges into 2 distinct clusters, an early composite zone $\alpha$, and a late prime zone $\beta$.

The prime zone $\beta$ features interesting crossing quasirays of prime $k$ arising from [A0], $p = \text{GPF}(i)$. Therefore we consider $i = mp : p = \text{GPF}(i)$ and see that the striations in $\beta$ attribute to $m$, with $p$ such that $m$ is small generally occurring earlier than those with larger $m$. Hence we refer to these as $\beta_m$. The "intertwining" quasirays in the prime zone are $\beta_3$ and $\beta_2$, with the latter usurping the former around $n = 2^{12}$.

The composite zone $\alpha$ generally arranges into conspicuous striations $\alpha_m$ resulting from [A1] where $m \mid k$. The lowest, "straggly" striation $\alpha_2$ that arises beginning with $a(29) = 8$ concerns $m = 2$. Striations and $\alpha_3$ and $\alpha_5$ show prominently rather early. A prominent early quasi-curve $\alpha_7$ starting with $a(13) = 14$ and extending to around $n = 93$ eventually seems to merge with $\alpha_5$ around that point. These four striations seem to prove to be principal features of scatterplot; no further striations seem to associate with prime $m$.

There are "echo" features in the earliest part of the graph for $n$ around 7000. For instance, we have $a(6956) = 10708$ through $a(6966) = 24093$ as a conspicuous example of echoing that takes place even for smaller $n$. In the range $n = 6956 \dots 6966$, we have alternating axioms, the early numbers arising from [A1] and coprimality.

Similar "echoes" appear between $a(8904) = 13444$ and $a(8914) = 30249$, and between $a(8988) = 13564$ and $a(8996) = 27128$.

## Conclusion.

This brief examined a lexically earliest sequence with a conditional function taking in three previous terms as input. Based on coprimality of $h$ and $j$, we set $a(n)$ to the smallest multiple of either the greatest prime factor of $i$ if true, else the smallest prime that does not divide $hj$. There are several simple consequences of definition, chief among them is the fact that [A0] implies $i \sqcup k$ and with lexical axiom, $i \neq k$, and that regarding both $hj$ versus $k$ and $i$ versus $k$, coprimality outside of given terms derives from [A1].

We have examined scatterplot to determine 2 principal zones, zone $\beta$ containing primes, and zone $\alpha$ primarily comprised of composites. Striations in zone $\beta$ pertain to $m : i = mp$ where $p = \text{GPF}(i)$, while those in zone $\alpha$ concern divisibility by 2, 3, 5, or 7.

Further study of "echo" features is warranted and may be appended to this paper in the future.

## References:


[1] N. J. A. Sloane, *The Online Encyclopedia of Integer Sequences*, retrieved November 2022.

[2] Michael Thomas De Vlieger, Constitutive Basics, *Simple Sequence Analysis*, 20230125.


## Code:

[C1] Generate S20230211:

```
nn = 2^20; c[_] = False; q[_] = 1;
Array[Set[{a[#], c[#]}, {#, True}] &, 3];
Set[{h, i, j}, {a[1], a[2], a[3]}]; u = 4;
Monitor[Do[If[CoprimeQ[h, j],
      (k = q[#]; While[c[k #], k++]; k *= #;
         While[c[# q[#]], q[#]++]) &[
         FactorInteger[i][[-1, 1]]],
      (k = q[#]; While[c[k #], k++]; k *= #;
         While[c[# q[#]], q[#]++]) &[(p = 2;
         While[Divisible[#, p], p = NextPrime[p]]; p) &[
         h j] ] ];
    Set[{a[n], c[k], h, i, j}, {k, True, i, j, k}];
    If[k == u, While[c[u], u++]], {n, 4, nn}], n];
  Array[a, nn] ];
```

## Concerns sequences:

A000040: Prime numbers.
A002110: Primorials.
A006530: Greatest prime factor $\text{GPF}(n)$.
A007947: Squarefree kernel of $n$; $\text{RAD}(n)$.
A053669: Smallest prime nondivisor of $n$.
A120944: "Varius" numbers; squarefree composites.
A126706: "Tantus" numbers neither prime power nor squarefree.
A246547: "Multus" numbers; composite prime powers $p^\varepsilon : \varepsilon \geq 1$.
A286708: "Plenus" numbers, products of multus numbers.

## Document Revision Record:

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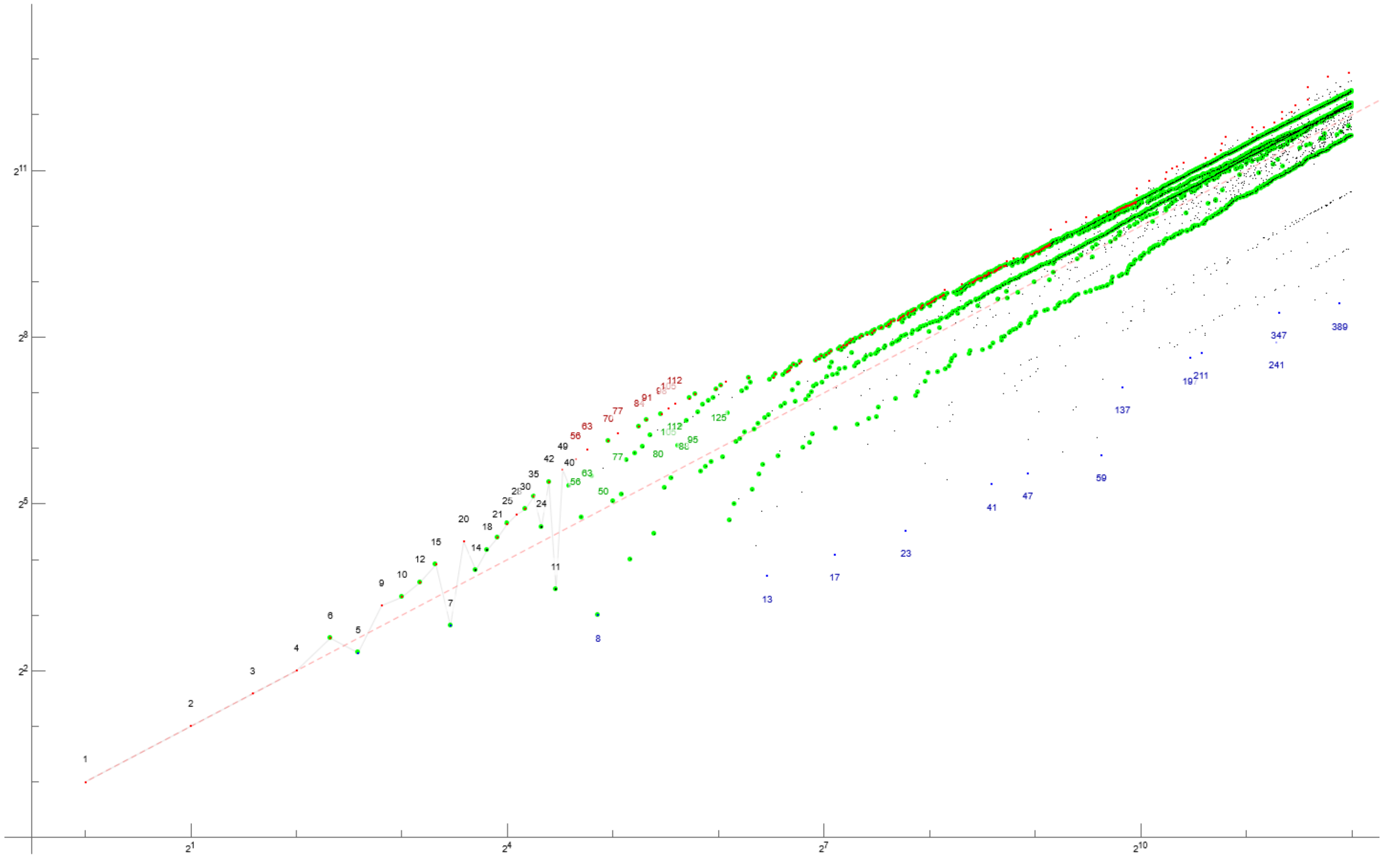

*Figure 3: Log log scatterplot of s20230211 for n = 1 … $2^{12}$, dashing in a pink line where a(n) = n, showing records in red, local minima in blue, and highlighting terms resulting from Axiom 1 in green.*

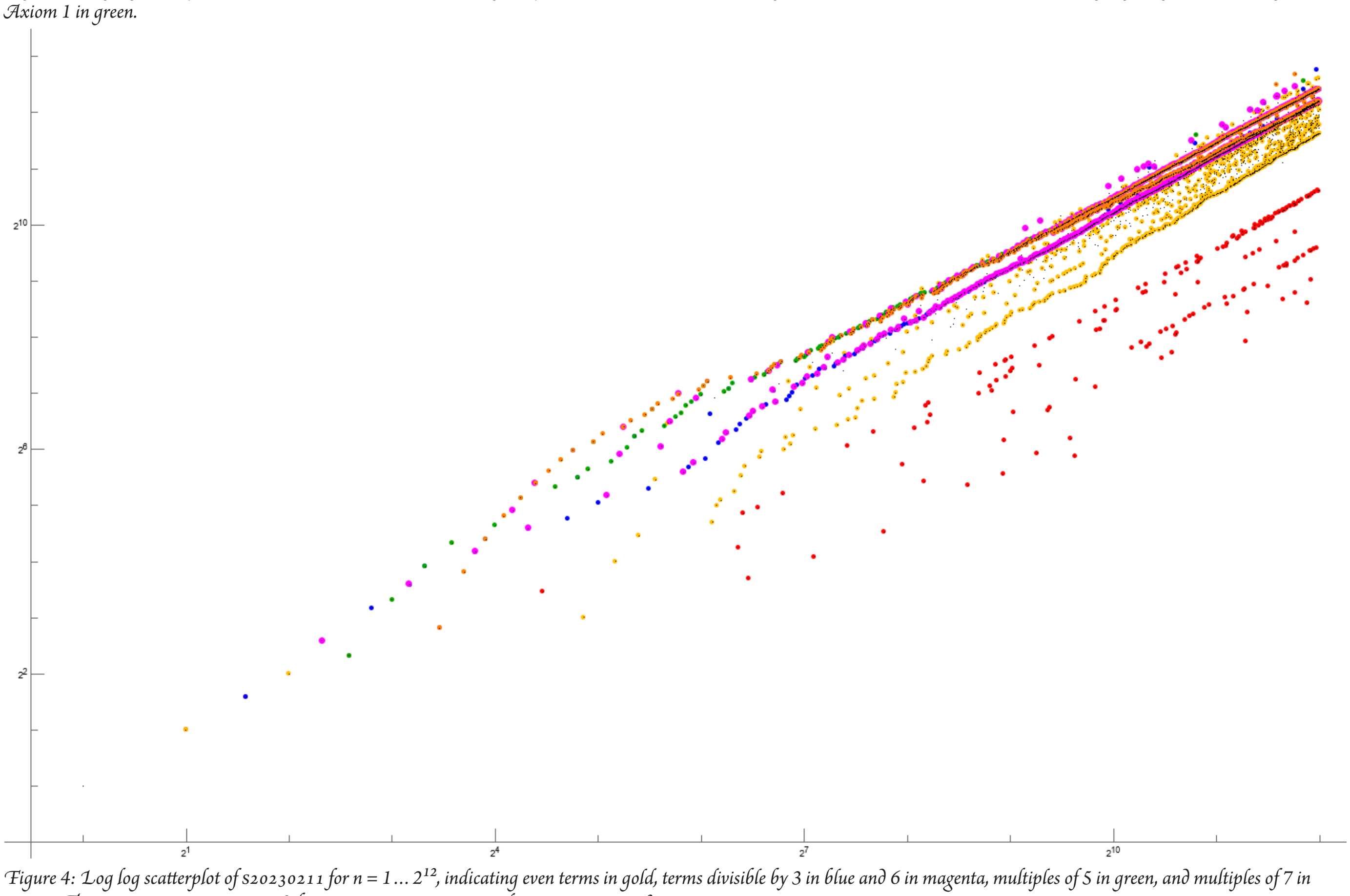

*Figure 4: Log log scatterplot of s20230211 for n = 1 … $2^{12}$, indicating even terms in gold, terms divisible by 3 in blue and 6 in magenta, multiples of 5 in green, and multiples of 7 in orange. These terms comprise zone α. Other prime terms appear in red, comprising zone β.*