On trying to exceed $\theta(\mathcal{P}(n+1))$ with $\theta(k)$ where k is prime(n) smooth.

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Abstract.

Primorials $\mathcal{P}(n)$ represent local minima of Euler's totient $\phi(n)$ and occur among local maxima of the regular counting function $\theta(n)$ = A010846(n). In the latter case, this has to do with the expansion of the bounded regular tensor in scope and rank. The least nondivisor prime q_1 has outsized impact on $\theta(n)$. Therefore we are led to consider a sequence of smallest PRIME(n) smooth k such that $\theta(k)$ is at least as large as $\theta(\mathcal{P}(n+1))$.

Introduction.

Consider k and n, nonzero positive integers. Here we are interested only in those $k \le n$. Recall the standard form prime power decomposition of n shown below.

$$n = \prod_{i=1}^{\omega} p_i^{\varepsilon_i}, \text{ prime } p \mid n,\ \omega = \omega(n). \qquad [1.1]$$

The empty product $n = 1$ is a product of no primes at all.

Let RAD(n) = A7947(n) = $\varkappa$ be the squarefree kernel of n as below:

$$\varkappa = \prod_{i=1}^{\omega} p_i, \text{ prime } p \mid n,\ \omega = \omega(n). \qquad [1.2]$$

We define an n-**regular** number k as k such that RAD(k) | n, that is, the squarefree kernel A7947(k) divides n.

We say k and n are **coregular** if RAD(k) = RAD(n) = $\varkappa$. From this, it is clear that $\omega(k) = \omega(n)$ for coregular k and n.

Since n-regularity depends on the squarefree kernel RAD(n) = $\varkappa$ independent of multiplicity, we then may generate a set $\boldsymbol{R}_\varkappa$ that contains all $\varkappa$-regular k, that is, the set of all numbers that are products of primes p such that $p \mid \varkappa$, raised to any nonnegative power ε:

$$\boldsymbol{R}_\varkappa = \{k : k \parallel \varkappa\}.$$
$$\boldsymbol{R}_\varkappa = \bigotimes_{p \mid \varkappa} \{p^\varepsilon : \varepsilon \ge 0\}. \qquad [1.3]$$

Therefore, the set $\boldsymbol{R}_\varkappa$ is the tensor product of prime divisor power ranges $\{p^\varepsilon : \varepsilon \ge 0\}$. The rank of $\boldsymbol{R}_\varkappa$ is $\omega(\varkappa)$. The cardinality of $\boldsymbol{R}_\varkappa$ is $\aleph_0$, since $|\{p^\varepsilon : \varepsilon \ge 0\}| = \aleph_0$ and, when sorted, we may assign an index i that makes the set countably infinite.

In the case of $\boldsymbol{R}_\varkappa$ where $\omega(\varkappa) = 1$, we simply have the prime power range for $\varkappa = p$, that is, $\{p^\varepsilon : \varepsilon \ge 0\}$. For example, $\boldsymbol{R}_2$ = A79.

An example of $\boldsymbol{R}_\varkappa$ for $\varkappa = 6$ is A3586 = $\boldsymbol{R}_6$ whose first terms follow:

```
1, 2, 3, 4, 6, 8, 9, 12, 16, 18, 24, 27, 32, 36, 48, 54,
64, 72, 81, 96, 108, 128, 144, 162, 192, 216, 243, 256,
288, 324, 384, 432, 486, 512, 576, 648, 729, 768, 864,
972, 1024, 1152, 1296, 1458, 1536, 1728, 1944, 2048, ...
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We may also write $\boldsymbol{R}_{12}$, $\boldsymbol{R}_{54}$, etc., but these subscripts ascribe to squarefree kernel $\varkappa = 6$, hence these are equivalent to $\boldsymbol{R}_6$.

If we are interested in coregular k such that RAD(k) = $\varkappa$, then we instead use the set $\varkappa\boldsymbol{R}_\varkappa$. Therefore, the set of k coregular to 6 is simply $6\boldsymbol{R}_6 = 6 \times$ A3586 which begins as follows:

```
6, 12, 18, 24, 36, 48, 54, 72, 96, 108, 144, 162, 192,
216, 288, 324, 384, 432, 486, 576, 648, 768, 864, 972,
1152, 1296, 1458, 1536, 1728, 1944, 2304, 2592, 2916,
3072, 3456, 3888, 4374, 4608, 5184, 5832, 6144, ...
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Since $\omega(6) = 2$, $6\boldsymbol{R}_6$ is simply A3586 stripped of prime powers. For $\varkappa\boldsymbol{R}_\varkappa$ with $\omega(\varkappa) > 2$, this is not true; $k \in \varkappa\boldsymbol{R}_\varkappa$ all have $\omega(k) = \omega(\varkappa)$, since by definition, all terms are divisible by $\varkappa$.

We are concerned in this work with k such that RAD(k) | n and $k \le n$. We denote this finite set $\check{\boldsymbol{R}}_n$ as follows:

$$\check{\boldsymbol{R}}_n = \{\, k : k \parallel \varkappa \wedge k \le n\}.$$
$$= \{\, k \in \bigotimes_{p \mid \varkappa} \{p^\varepsilon : \varepsilon \ge 0\} : \wedge\, k \le n\,\}. \qquad [1.4]$$

We write the subscript n rather than squarefree kernel $\varkappa$ to specify the discrete limit. Then $\mathbf{C}_n$ is the set containing those $k = m\varkappa$ not exceeding n where m is $\varkappa$-regular. Simply, $\mathbf{C}_n$ contains $k \le n$ coregular to $\varkappa$ = RAD(n).

$$\mathbf{C}_n = \{\, k : k = m\varkappa \wedge m \parallel \varkappa \wedge m\varkappa \le n\,\}. \qquad [1.5]$$

Therefore, for $n = 12$, we have the following:

$$\check{\boldsymbol{R}}_{12} = \{1, 2, 3, 4, 6, 8, 9, 12\},$$
$$\mathbf{C}_{12} = \{6, 12\}.$$

The Regular Counting Function.

This section introduces basics about the regular counting function $\theta(n)$ = A010846(n) and its relation to the divisor counting function $\tau(n)$ = A5(n).

Define the regular counting function as follows:

$$\theta(n) = |\{k : k \parallel \varkappa \wedge k \le n\,\}|$$
$$= |\check{\boldsymbol{R}}_n|$$
$$= \text{A010846}(n). \qquad [2.1]$$

Let us examine the divisor counting function.

$$\text{For } n = \prod_{i=1}^{\omega} p_i^{\varepsilon_i}, \text{ prime } p \mid n,\ \omega = \omega(n),$$

$$\tau(n) = \prod_{i=1}^{\omega} (\varepsilon_i + 1) \qquad [2.2]$$

$$= \bigotimes_{i=1}^{\omega} \{p_i^{\delta_i} : \delta_i = 0 \ldots \varepsilon_i\}. \qquad [2.3]$$

Example: for $n = 12 = 2^2 \times 3$, $\tau(12) = (2+1)(1+1) = 2 \times 3 = 6$ via [2.2]. A diagram of the outer product approach [2.3] appears below:

	2^0	2^1	2^2
3^0	**1**	**2**	**4**
3^1	**3**	**6**	**12**

[2.4]

The outer product approach lends insight toward an algorithm we can employ to most efficiently construct a table of divisors of n. In Table [1.9], we see the following:

$$\tau(2^2 \times 3) = \{2^\delta : \delta = 0 \ldots 2\} \otimes \{3^\delta : \delta = 0 \ldots 1\}$$
$$= \{1, 2, 4\} \otimes \{1, 3\}$$
$$= \{\{1, 2, 4\}, \{3, 6, 12\}\}$$
$$= \text{row 12 of A275055}. \qquad [2.5]$$

The sequence A275055 lists divisors in the order of appearance read left to right, then by level, etc. through all ranks of $\check{\boldsymbol{R}}_n$, hence the row is vectorized to {1, 2, 4, 3, 6, 12}. We compare this to row 12 of A162306 = {1, 2, 3, 4, 6, 12}, where we regard the operation $\otimes$ instead as a Kronecker product.

Now consider $n = 60$, with $\omega(60) = 3$. The outer product approach toward a table of divisors of 60 appears in [2.6]. Compare this to $\check{\boldsymbol{R}}_{60}$, that is, the set of numbers $k \le 60$ that are also regular to 60, which is shown in Figure [2.7].

$\times 5^0$	2^0	2^1	2^2
3^0	**1**	**2**	**4**
3^1	**3**	**6**	**12**

$\times 5^1$	2^0	2^1	2^2
3^0	**5**	**10**	**20**
3^1	**15**	**30**	**60**

[2.6]

1	**2**	**4**	8	16	32
3	**6**	**12**	24	48	
9	18	36			
27	54				

5	**10**	**20**	40
15	**30**	**60**	
45			

25	50

[2.7]

The finite $\omega(\varkappa)$-rank tensor $\check{\boldsymbol{R}}_n$ derives from infinite $\boldsymbol{R}_\varkappa$ bounded by n. The divisor tensor $\boldsymbol{D}$ is the product of power ranges $\{\, p^\varepsilon : p^\varepsilon \mid n \,\}$, while the regular tensor $\check{\boldsymbol{R}}_n$ is the product, bounded by n, of power ranges $\{\, p^\varepsilon : p \mid n \wedge\ p^\varepsilon \le n \,\}$. The former, $\boldsymbol{D}$, is an orthogonal array set within $\check{\boldsymbol{R}}_n$. The finite $\omega(\varkappa)$-rank tensor $\check{\boldsymbol{R}}_n$ involves an irregular discrete "surface" or "sheet"; both are contained in $\boldsymbol{R}_\varkappa$. The geometry of $\check{\boldsymbol{R}}_n$ approximates an $\omega(\varkappa)$-dimensional orthogonal simplex ($\omega(\varkappa)$-orthosimplex) with an origin-antipodal $\omega(n{-}1)$-simplex facet that joins the largest values of all distinct prime divisor power ranges bounded by n. The geometry of the orthosimplex may be amenable to calculus and is beyond the scope of this paper.

In brief, [2.6] when vectorized as generated, is merely row 60 of A275055 while row 60 of A027750 is the same set of divisors, sorted. Table [2.7] is what we obtain when we allow an algorithm to iterate the exponent of a prime power factor until the product exceeds n. This constructs $\check{\boldsymbol{R}}_n$ efficiently to yield row n of A275280 in a manner analogous to [2.6]. The algorithm in question appears in the pseudocode below:

```
let n = 60;
for (i = 0, i ≤ ⌊log₂ n⌋ ) {
    for (j = 0, j ≤ ⌊log₃ n/2^i⌋ ) {
        for (k = 0, k ≤ ⌊log₅ n/(2^i × 3^j)⌋ ) {
            2^a × 3^b × 5^c } } }
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[2.8]

Mertens-Like Regular Counting Function

There are several methods for computing $\theta(n)$. Notably, Benoit Cloitre [1: A010846] shows that we may employ the reduced residue system (RRS) of n, where totative t such that $(t, n) = 1$ (i.e., $t \perp n$) in the following summation:

$$\theta(n) = \sum_{t<n}^{t\perp n} \mu(t) \times \lfloor n/t \rfloor. \qquad [2.9]$$

where $\mu(n)$ is the Möbius function of n. This summation links the regular counting function with the totative counting function, better known as the Euler totient function.

Define $\check{\boldsymbol{T}}_n$ to be the RRS of n, the set of $1 \le t < n$ such that $(t, n) = 1$.

$$\check{\boldsymbol{T}}_n = \{\, t : (t, n) = 1 \wedge t < n \,\}. \qquad [2.10]$$

$$\boldsymbol{T}_n = \{\, mt : (t, n) = 1 \wedge t < n \wedge m \ge 1 \,\}. \qquad [2.11]$$

where [2.11] is the set of numbers coprime to n, tantamount to the set of numbers coprime to $\varkappa = \text{RAD}(n)$.

The Euler totient function is the cardinality of $\check{\boldsymbol{T}}_n$ shown below:

$$\begin{aligned}\phi(n) &= |\,\check{\boldsymbol{T}}_n\,| \\ &= n \prod_{p \mid n} (1 - 1/p)\end{aligned} \qquad [2.12]$$

We note that $\check{\boldsymbol{R}}_n \cap \check{\boldsymbol{T}}_n = \boldsymbol{R}_\varkappa \cap \boldsymbol{T}_\varkappa = \{1\}$, which makes the construction of $\theta(n)$ via [2.9] interesting.

Define a **primorial** to be a product of the smallest n primes:

$$\mathcal{P}(n) = \text{A2110}(n) = \text{V0111}(n)\ \prod_{i=1}^{n} \text{PRIME}(n). \qquad [3.1]$$

We are interested in primorials $\mathcal{P}(n)$ since they minimize the totient ratio $\phi(n)/n$ and represent local minima for $\phi(n)$, while they occur among local maxima for $\theta(n)$.

The Möbius function method of generating $\theta(n)$ in [2.9] merits examination not merely because it differs from the "intuitive" methodologies associated with the properties of n-regular k themselves, but because of implications regarding the smallest primes q coprime to n. Chief among the implications is that small prime totatives wreak havoc against a high value of $\theta(n)$.

Define function $f(n, t)$ as follows:

$$\begin{aligned} f(n, t) &= \mu(t) \times \lfloor n/t \rfloor, \\ \text{with } (n, t) &= 0,\ t < n. \end{aligned} \qquad [3.2]$$

Let q_1 be the least nondivisor prime of n, i.e, $q_1 = \text{LNP}(n) = \text{A053669}(n)$, and generally, let q be a prime does not divide n. We can determine the following about the behavior of the function f. The value of $f(n, q)$ applied to prime $q < ½\, n$ is negative with an absolute value greater than 1. The absolute value is most pronounced for $q_1 = 2$ and decreases as q increases. For prime $q > ½\, n$ the value is -1.

Consider $\boldsymbol{R}_{\mathcal{P}(n)}$, the infinite set of k regular to $\mathcal{P}(n)$. It is clear that $\boldsymbol{R}_{\mathcal{P}(n)}$ is the set of PRIME(n)-smooth numbers. Therefore $\check{\boldsymbol{R}}_{\mathcal{P}(n)}$ is the set of PRIME(n)-smooth numbers $k \le \mathcal{P}(n)$.

The empty product is the smallest number coprime to n. The value of $f(n, 1) = n$, since $n/1 = 1$ and $\mu(1) = 1$. Hence, beginning with $t = n$, $\theta(n) = n$, with subsequent $f(n, t)$ for $t > 1$ modifying the value to arrive at actual $\theta(n)$.

Those totatives $t < ½\, n$ have the greatest effect on the ultimate value of $\theta(n)$ for the following reasons:

1. The totatives of n are symmetrically arranged about $½\, n$. In other words, $t < n$ such that $(t, n) = 1$ implies $(n, n - t) = 1$.
2. $\lfloor n/t \rfloor > 1$ for $t < ½\, n$ while $\lfloor n/t \rfloor = 1$ for $t < ½\, n$.
3. $\lfloor n/q_1 \rfloor$ is maximal since q_1 is the smallest prime that is coprime to n.
4. Let S = the sum of $f(n, t)$ across $½\, n < t < n$. Then $f(n, q_1) \ge S$. The set of numbers that have $f(n, q_1) = S$ is finite: $\{3, 4, 6, 8, 12, 18, 24, 30\}$, cf. A048597.

Hence, the least nondivisor prime q_1 has the most influence on $\theta(n)$. This supports interest in $\check{\boldsymbol{R}}_{\mathcal{P}(n)}$.

Examination of q_1 alone is incomplete regarding the full effect of the smallest prime totative q_1 on $\theta(n)$.

Given [3.1], the following is evident regarding primorial $\mathcal{P}(n)$:

$$\begin{aligned} p_n &< q_1, \text{ that is,} \\ \text{GPF}(\mathcal{P}(n)) &< \text{LNP}(\mathcal{P}(n)). \end{aligned} \qquad [3.3]$$

In this way, $\mathcal{P}(n)$ maximizes q_1 for numbers m with $\omega(m) = n$.

For example, regarding squarefree numbers $\varkappa$ such that $\omega(\varkappa) = 3$, the smallest such number is $\mathcal{P}(3) = 30 = 2 \times 3 \times 5$, where $q_1 = 7$. Any other three prime factors produces a number that has $q_1 < 7$. Therefore, among $\varkappa$ such that $\omega(\varkappa) = 3$, primorial $\mathcal{P}(3) = 30$ maximizes q_1.

Let's consider the effect of multiplication on the tensor $\check{\boldsymbol{R}}_n$. A prime number must either divide or be coprime to n. Therefore we consider the following cases.

Case 1 involves pn, where prime $p \mid n$. Since p by definition is prime, $pn > n$. It is clear that coregular k and n share the same squarefree kernel $\varkappa$ and therefore $\boldsymbol{R}_\varkappa$. It follows that both $\text{RAD}(n) = \text{RAD}(pn) = \varkappa$ and $\omega(n) = \omega(pn)$.

If $n = p^{\varepsilon}Q$, then it is obvious $pn = p^{(\varepsilon+1)}Q$, hence, $(\varepsilon+1) > 1$. No new prime divisors are introduced and none are lost. Since $\omega(n) = \omega(pn)$, the rank of $\boldsymbol{R}_{\varkappa}$ is maintained. These facts together imply the new tensor $\check{\boldsymbol{R}}_{pn}$ is merely $\boldsymbol{R}_{\varkappa}$ bounded not by n, but by $pn > n$.

Therefore, multiplication of n by a prime $p \mid n$ merely increases the bound and admits more regular terms from the infinite regular set $\boldsymbol{R}_{\varkappa}$ common to both n and pn. Hence, $\theta(pn) > \theta(n)$.

<u>Case 2</u> involves qn, with prime q coprime to n. Since q is prime by definition, the product $qn > n$. The set $\boldsymbol{R}_{\varkappa}$ cannot be that of qn, since the distinct prime factors of qn have an additional prime factor q that is missing in n, i.e., $\mathrm{RAD}(n) < \mathrm{RAD}(qn)$. Furthermore, $\omega(qn) = \omega(n) + 1$, implying $\boldsymbol{R}_{q\varkappa}$ has greater rank than $\boldsymbol{R}_{\varkappa}$. Specifically, $\boldsymbol{R}_{q\varkappa}$ has 1 more dimension than $\boldsymbol{R}_{\varkappa}$ via the following tensor product:

$$\boldsymbol{R}_{q\varkappa} = \boldsymbol{R}_{\varkappa} \otimes \{\, q^{\varepsilon} : \varepsilon \geq 0 \,\}. \qquad [4.1]$$

Furthermore, since $qn > n$, $\check{\boldsymbol{R}}_{q\varkappa}$ is bounded at a larger value qn. Therefore $\theta(qn)$ must be significantly larger than $\theta(n)$.

<u>Case 3</u>. Suppose we want to conserve the value of $\omega(n)$. Case 1 above conforms to such conservation, while Case 2 violates it. We can, however, "bargain away" prime p such that $p \mid n$ for a nondivisor prime q so as to obtain qn/p. This way, the number of distinct prime factors is conserved, i.e., $\omega(qn/p) = \omega(n)$, and the regular tensors of both n and qn/p have the same rank.

Now suppose $n = \mathcal{P}(\tau)$ and that $q_1 = \mathrm{LNP}(\mathcal{P}(\tau))$. Here are some implications:

1. $q > \mathrm{GPF}(\mathcal{P}(\tau))$, i.e., $q > p_{\tau}$.
2. $qn/p > n$, since $q > p$ for $p \mid \mathcal{P}(\tau)$.

Let $q_1 = \mathrm{PRIME}(\tau+1)$. Then $q_1 > \mathrm{LNP}(qn/p)$ which we can rewrite as simply $q_1 > p$.

Since $p < q_1$ and $q_1 n/p > n$, $\lfloor n/p \rfloor > \lfloor n/q_1 \rfloor$. This implies that, although $\theta(q_1 n/p)$ may exceed $\theta(n)$, $q_1 n/p$ certainly is less efficient at producing regulars; indeed the magnitude of the larger number might only be overcoming the reduced efficiency. Thus we may see certain numbers like $q_1 n/p$ among the terms of A244052, following $\mathcal{P}(\tau)$ in the sequence.

How Early does $\mathcal{P}(n+1)$ appear in A244052?

What is the smallest $\mathrm{PRIME}(n)$-smooth k such that $\theta(k)$ is no smaller than $\theta(\mathcal{P}(n+1))$? The question is motivated by Cases 1 and 2 above. Therefore we pose the sequence A363794, defined as follows:

$$\begin{aligned} a(n) &= \boxtimes k,\ k \in \boldsymbol{R}_{\mathcal{P}(n)},\ \theta(k) \geq \theta(\mathcal{P}(n+1)) \\ &= \boxtimes k,\ k \in \boldsymbol{R}_{\mathcal{P}(n)},\ \mathrm{A010846}(k) \geq \mathrm{A010846}(\mathrm{A2110}(n+1)) \\ &= \boxtimes k,\ k = \boldsymbol{R}_{\mathcal{P}(n)}(i),\ i \geq \mathrm{A363061}(n+1). \qquad [5.1] \end{aligned}$$

Hence, the question is tantamount to finding the smallest index $i \geq \mathrm{A363061}(n+1)$ such that $\boldsymbol{R}_{\mathcal{P}(n)}(i)$ is coregular to $\mathcal{P}(n)$. It follows from Case 2 that $\boldsymbol{R}_{\mathcal{P}(n)}(i) > \mathcal{P}(n+1)$. The sequence demonstrates the efficiency of $\mathcal{P}(n+1)$ over $\mathrm{PRIME}(n)$-smooth m in generating regular numbers $k \leq n$.

The first terms of A363794 are as follows:

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16, 72, 540, 6300, 92400, 1681680, 36756720, 921470550,
27886608750, 970453984500, 37905932634570, ...
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Table [5.2] below shows n, $\mathrm{PRIME}(n)$, and primorial $\mathcal{P}(n+1)$ in the first 3 columns, respectively. The fourth column gives the term $a(n)$ = A363794(n), followed by A363061$(n) = \theta(\mathcal{P}(n+1))$ and $\theta(a(n))$. The last two columns show $a(n) = \mathbf{C}(j) = m \times \mathcal{P}(n)$.

n	p(n)	P(n+1)	a(n)	θ(P(n))	θ(a(n))	j	m
1	2	6	16	5	5	4	8
2	3	30	72	18	18	8	12
3	5	210	540	68	69	13	18
4	7	2310	6300	283	290	22	30
5	11	30030	92400	1161	1165	29	40
6	13	510510	1681680	4843	4848	42	56
7	17	9699690	36756720	19985	19994	53	72
8	19	223092870	921470550	83074	83435	68	95
9	23	6469693230	27886608750	349670	351047	89	125
10	29	200560490130	970453984500	1456458	1457926	107	150

This data shows that $a(n)$ is several times $\mathcal{P}(n+1)$. As for records in $\theta(n)$, (i.e., highly regular numbers A244052), we see that $\mathcal{P}(n+1)$ appears long before $\mathrm{PRIME}(n)$-smooth number appears that has more regular numbers.

Conclusion.

We have examined the regular counting function $\theta(n)$ = A010846(n) and have discerned that primorials $\mathcal{P}(n)$ set records and therefore are highly regular numbers in A244052. Since primorials also represent local minima in Euler's totient $\phi(n)$, we are drawn to what might make them relevant to such apparently disparate constitutive counting functions.

We have shown that the least nondivisor prime q_1 presents an outsized impact against the number $\theta(n)$, through the lens of a formula for $\theta(n)$ involving $\mu(t) \times \lfloor n/t \rfloor$. We also examined the effect of multiplication of n by both prime divisor $p \mid n$ and nondivisor prime q coprime to n. Specifically, the latter product presents a bounded regular tensor $\check{\boldsymbol{R}}$ with greater rank, therefore, increased density of n-regular k such that $k \leq n$.

This leads us to consider the smallest $\mathrm{PRIME}(n)$-smooth k that has at least as many regular numbers (no greater than itself) as does $\mathcal{P}(n+1)$. Given the first few terms, it is evident that $k > \mathcal{P}(n+1)$ by a significant factor.

References:

[1] N. J. A. Sloane, *The Online Encyclopedia of Integer Sequences*, retrieved June 2023.

[2] Michael Thomas De Vlieger, Constitutive Counting Functions for Primorials, *Simple Sequence Analysis*, 20230621.

Concerns sequences:

A000005: Divisor counting function $\tau(n)$.
A000010: Euler totient function $\phi(n)$.
A000040: Prime numbers.
A002110: Primorials $\mathcal{P}(n)$.
A010846: Regular counting function $\theta(n)$.
A038566: List of n-totatives $\check{T}_n = \{\, k : k \perp n \wedge k < n \,\}$.
A053669: Smallest prime that does not divide n.
A363061: $\theta(\mathcal{P}(n)) = \mathrm{A010846}(\mathrm{A2110}(n))$.
A363844: $\xi_t(\mathcal{P}(n)) = \mathrm{A243823}(\mathrm{A2110}(n))$.
A363794: Least prime(n) smooth k such that $\theta(k) \geq \mathrm{A363061}(n)$.

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