Study of Balance between Divisors and Totatives of Integral Number Bases r

The following diagrams document observations of the relationships among divisors and totatives of integers between two and two and a half dozen, inclusive, and the third, fourth, and fifth multiples of the dozen.

The divisors of an integer *r*, under consideration as a number base, can be regarded as "leverage", tools which one might use in manipulating numeric data, especially through the operations of multiplication and division. Each divisor *d* of base *r* has a multiplicative reciprocal *d'* such that an equation $d \times d' = r$ can be written. The "leverage" of base *r* can be measured by dividing the sum of all divisors σ_0 by the base r (σ_0 / r); the greater the "leverage", the greater the effectiveness of the divisor set of a number base for consideration as a base of general computation.

The totatives of an integer, those digits of a base *r* which are relatively prime to r, represent resistance to this leverage. These totatives are either primes smaller than *r* which do not appear in the prime factorization of *r* (unrepresented primes or UP) or they are composed of unrepresented primes (products of unrepresented primes or PUP). Each totative *t* of base *r* has an additive reciprocal *t'* such that an equation $t + t' = r$ can be written. The "resistance" of base *r* can be measured by dividing the sum of all totatives (the Euler totient function) φ by the base *r* (φ / r). The greater the "resistance", the more blunted the usefulness of a number base for general computation.

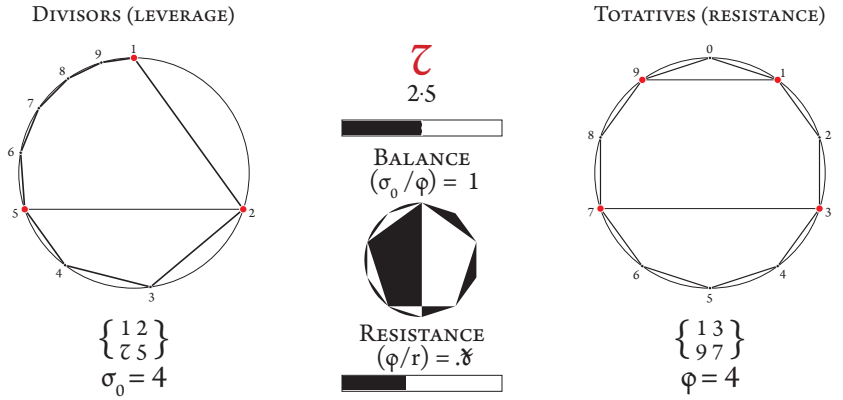
"Balance" between leverage and resistance is computed by dividing the sum of all divisors by the sum of all totatives (σ_0 / φ). This equation yields divisor dominance only for bases 2, 4, 6, and 12. Parity between leverage and resistance is afforded by bases 3, 8, 10, 18, 24, and 30, with the last figure being the largest integral number base to feature a 1-to-1 or better balance between leverage and resistance. All greater numbers and those not mentioned feature totative dominance; more resistance than leverage.

The diagrams illustrate the relationships among the divisors on the left, and the totatives on the right. The divisor diagram logarithmically arranges the digits *n* of base *r* around a circle, placing 1 and *r* at the top, and running clockwise. Divisor pairs appear horizontally across from one another. In the example below, the divisors 2 and 5 are joined by a horizontal line. In the case of bases *r* which are powers of a digit $r^{1/x}$, an *x*-sided regular polygon joins the powers of the digit $r^{1/x}$. The set of divisors *D* of *r* appears below the figure, the pairs arranged vertically within the braces. The sum of divisors σ_0 appears below this.

The totative diagram arranges the digits *n* of base *r* around a circle at equally distant points, dividing the circle in *r* segments, running clockwise. Totative pairs appear horizontally across from one another. In the example below, the totatives in each pair {1, 3} and {7, 9} are joined by horizontal lines. The set of totatives *T* of *r* appears below the figure, the pairs arranged vertically within the braces. The sum of totatives appears below this.

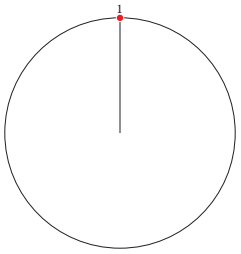
In the center, the argam numeral and prime composition of the number r is shown, along with a cyclical diagram. The "balance" ratio appears above the diagram, while the "resistance" ratio appears below.

All the figures in this study attempt to visually analyze positive integers for consideration as a number base for general human computation.



2

2



$\left\{\begin{matrix}1\\2\end{matrix}\right\}$

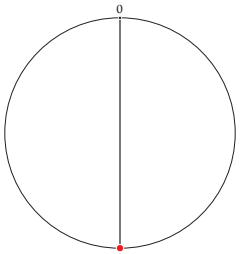
$\sigma_0 = 2$

Balance

$(\sigma_0/\varphi) = 2$

Resistance

$(\varphi/r) = .\mathcal{Z}$

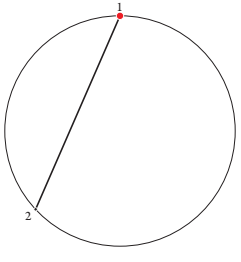


$\left\{1\right\}$

$\varphi = 1$

3

3



$\left\{\begin{matrix}1\\3\end{matrix}\right\}$

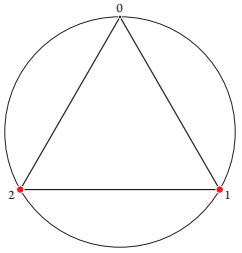
$\sigma_0 = 2$

Balance

$(\sigma_0/\varphi) = .\mathcal{Z}$

Resistance

$(\varphi/r) = .\mathcal{Y}$

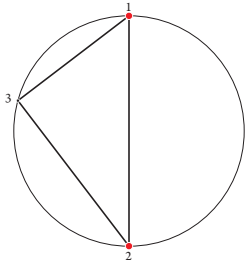


$\left\{\begin{matrix}1\\2\end{matrix}\right\}$

$\varphi = 2$

4

2^2



$\left\{\begin{matrix}1&2\\4\end{matrix}\right\}$

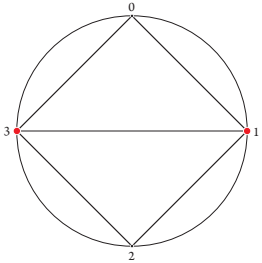
$\sigma_0 = 3$

Balance

$(\sigma_0/\varphi) = 1.\mathcal{Z}$

Resistance

$(\varphi/r) = .\mathcal{Z}$

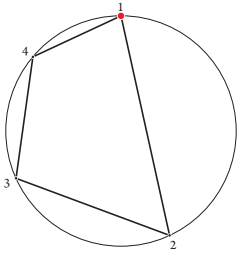


$\left\{\begin{matrix}1\\3\end{matrix}\right\}$

$\varphi = 2$

5

5



$\left\{\begin{matrix}1\\5\end{matrix}\right\}$

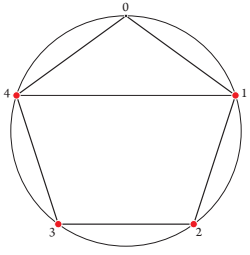
$\sigma_0 = 2$

Balance

$(\sigma_0/\varphi) = .\mathcal{Z}$

Resistance

$(\varphi/r) = .\mathcal{X}$

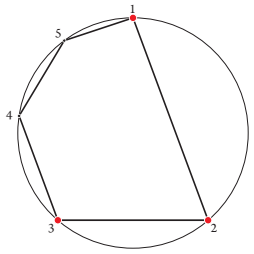


$\left\{\begin{matrix}1&2\\5&4\end{matrix}\right\}$

$\varphi = 4$

6

$2\cdot3$



$\left\{\begin{matrix}1&2\\6&3\end{matrix}\right\}$

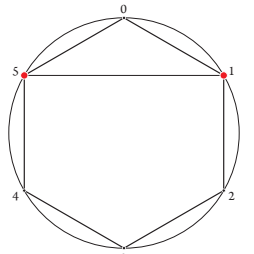
$\sigma_0 = 4$

Balance

$(\sigma_0/\varphi) = 2$

Resistance

$(\varphi/r) = .\mathcal{D}$

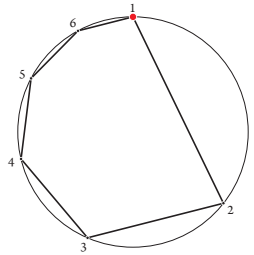


$\left\{\begin{matrix}1\\5\end{matrix}\right\}$

$\varphi = 2$

7

7



$\left\{\begin{matrix}1\\7\end{matrix}\right\}$

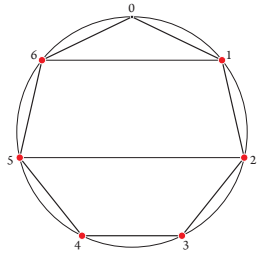
$\sigma_0 = 2$

Balance

$(\sigma_0/\varphi) = .\mathcal{Z}$

Resistance

$(\varphi/r) = .\mathcal{ƐƧƆ}$

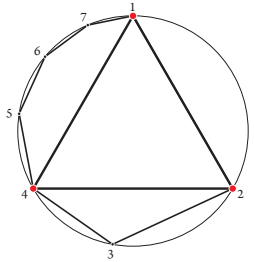


$\left\{\begin{matrix}1&2&3\\6&5&4\end{matrix}\right\}$

$\varphi = 6$

8

2^3



$\left\{\begin{matrix}1&2\\8&4\end{matrix}\right\}$

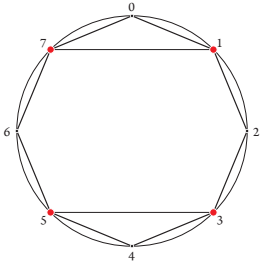
$\sigma_0 = 4$

Balance

$(\sigma_0/\varphi) = 1$

Resistance

$(\varphi/r) = .\mathcal{Z}$

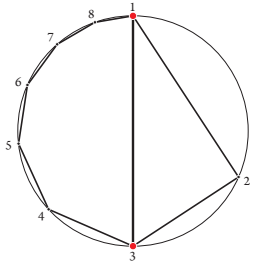


$\left\{\begin{matrix}1&3\\7&5\end{matrix}\right\}$

$\varphi = 4$

9

3^2



$\left\{\begin{matrix}1\\9&3\end{matrix}\right\}$

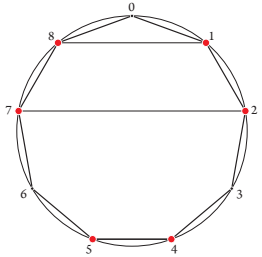
$\sigma_0 = 3$

Balance

$(\sigma_0/\varphi) = .\mathcal{Z}$

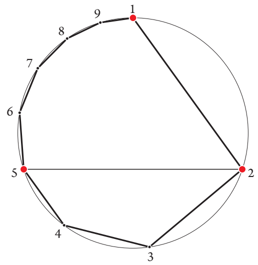
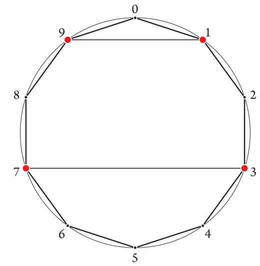
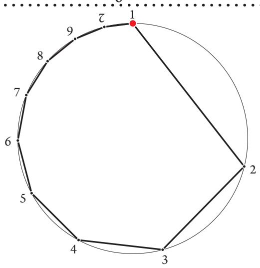
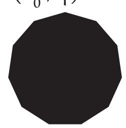
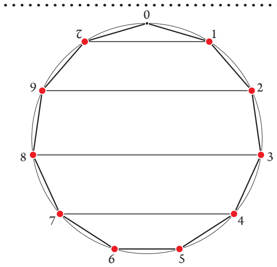
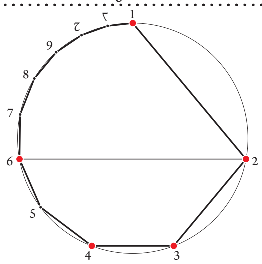
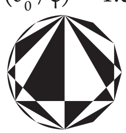
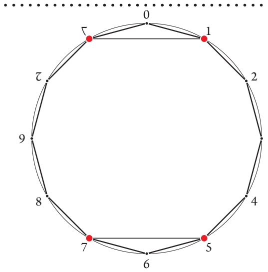
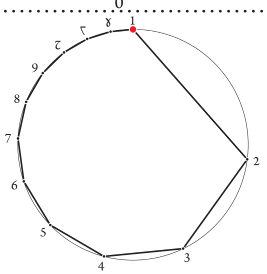
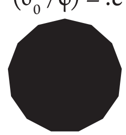
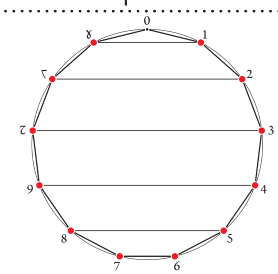
Resistance

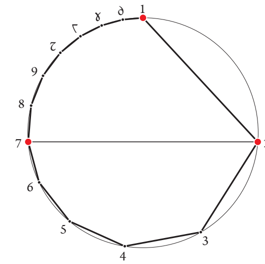
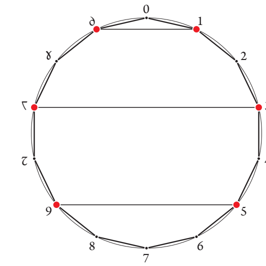
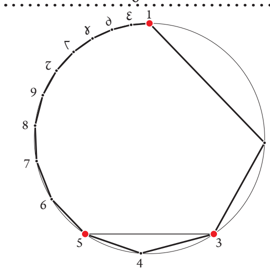
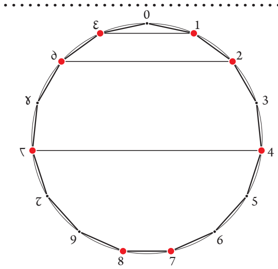
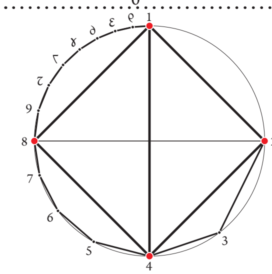
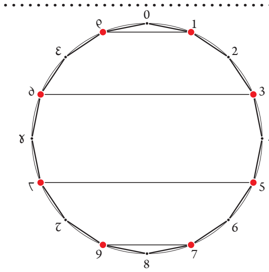
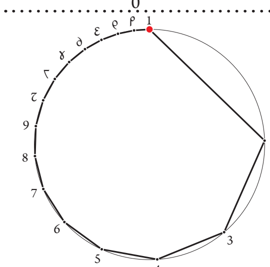
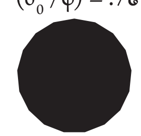
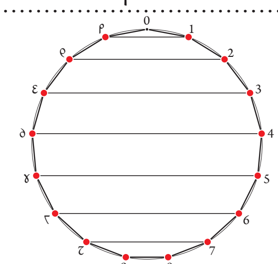
$(\varphi/r) = .\mathcal{Y}$

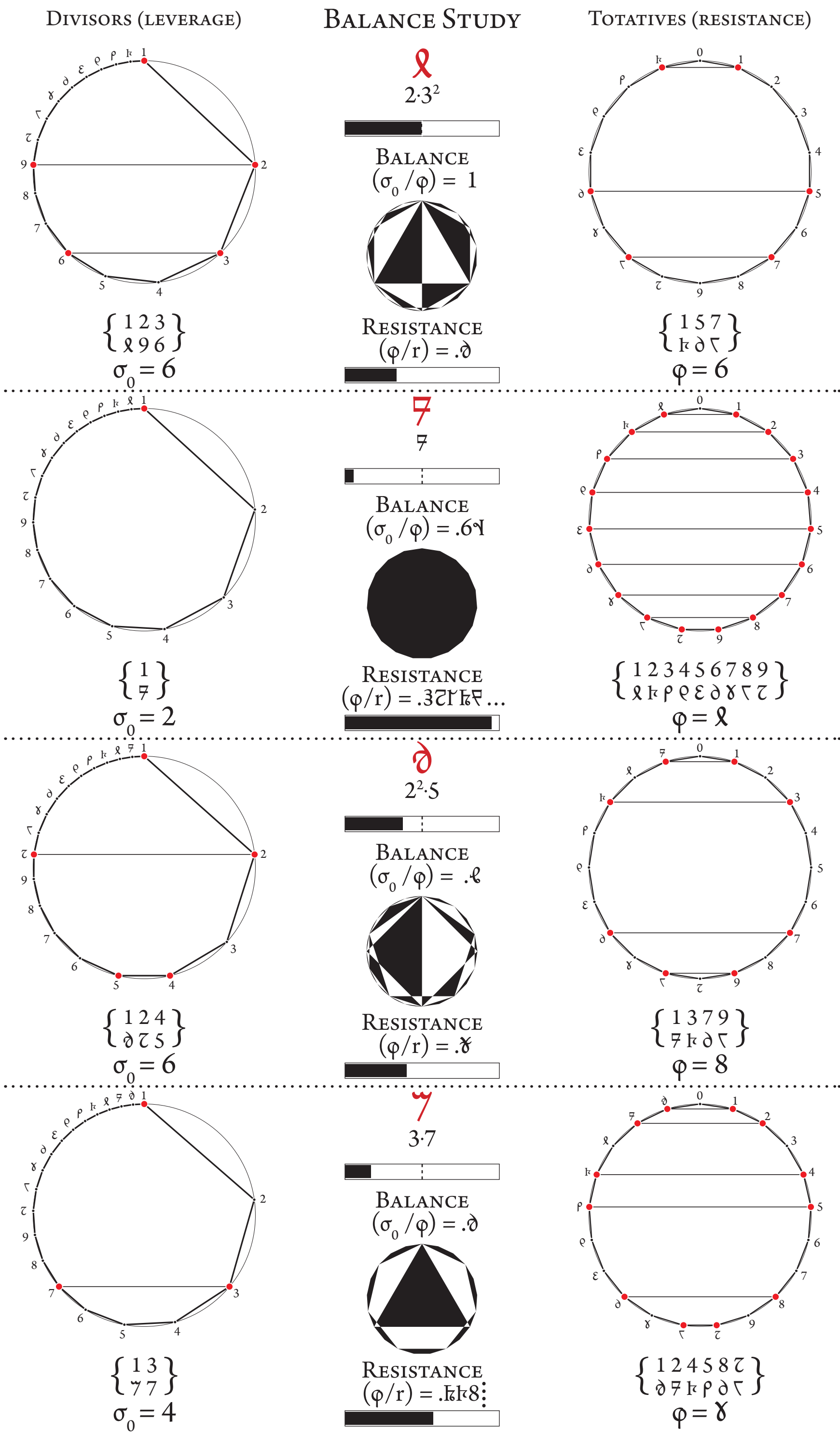


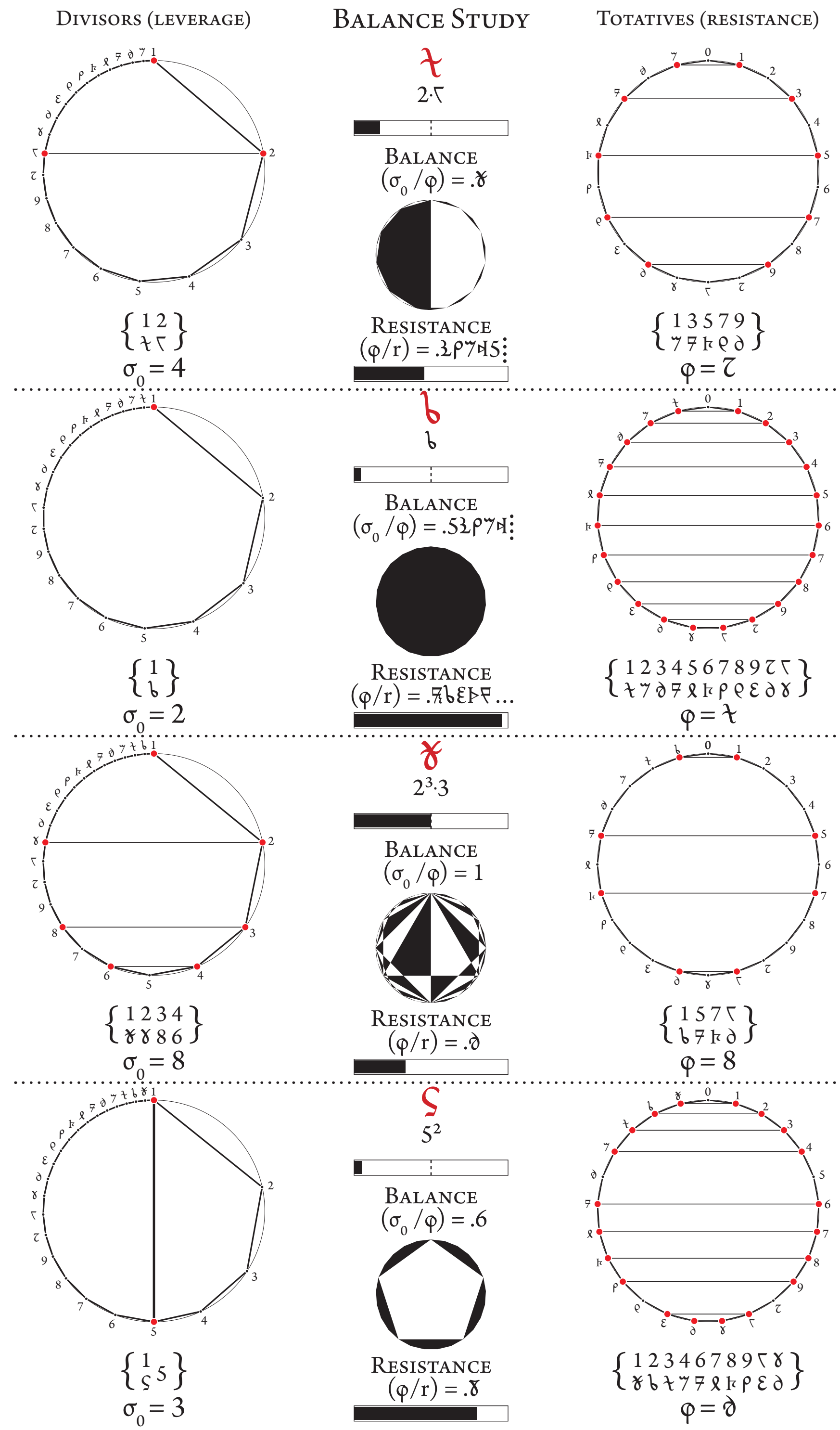
$\left\{\begin{matrix}1&2&4\\8&7&5\end{matrix}\right\}$

$\varphi = 6$

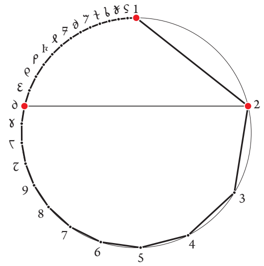
Divisors (leverage)	Balance Study	Totatives (resistance)
 {1 2 / Շ 5} $\sigma_0 = 4$	**Շ** 2·5  Balance $(\sigma_0/\varphi) = 1$ Resistance $(\varphi/r) =$.ɣ	 {1 3 / 9 7} $\varphi = 4$
 {1 / ꞁ} $\sigma_0 = 2$	**ꞁ** ꞁ  Balance $(\sigma_0/\varphi) =$.ɣ Resistance $(\varphi/r) =$.Ƨ8ᘔƐՇ⋮	 {1 2 3 4 5 / Շ 9 8 7 6} $\varphi =$ Շ
 {1 2 3 / ɣ 6 4} $\sigma_0 = 6$	**ɣ** $2^2 \cdot 3$  Balance $(\sigma_0/\varphi) =$ 1.Ƨ Resistance $(\varphi/r) =$.∂	 {1 5 / ꞁ 7} $\varphi = 4$
 {1 / ∂} $\sigma_0 = 2$	**∂** ∂  Balance $(\sigma_0/\varphi) =$.Շ Resistance $(\varphi/r) =$.Ƨ649⋮	 {1 2 3 4 5 6 / ɣ ꞁ Շ 9 8 7} $\varphi =$ ɣ

Divisors (leverage)	Balance Study	Totatives (resistance)
 {1 2 / Ɛ 7} $\sigma_0 = 4$	**Ɛ** 2·7  Balance $(\sigma_0/\varphi) =$.Ч Resistance $(\varphi/r) =$.Ƨ∂ꝁ⋮	 {1 3 5 / ∂ ꞁ 9} $\varphi = 6$
 {1 3 / ϱ 5} $\sigma_0 = 4$	**ϱ** 3·5  Balance $(\sigma_0/\varphi) =$.Ƨ Resistance $(\varphi/r) =$ .8	 {1 2 4 7 / Ɛ ∂ ꞁ 8} $\varphi = 8$
 {1 2 4 / ρ 8} $\sigma_0 = 5$	**ρ** 2^4  Balance $(\sigma_0/\varphi) =$.ꝊƧ Resistance $(\varphi/r) =$.Ƨ	 {1 3 5 7 / ϱ ∂ ꞁ 9} $\varphi = 8$
 {1 / ꝁ} $\sigma_0 = 2$	**ꝁ** ꝁ  Balance $(\sigma_0/\varphi) =$ .7Ƨ Resistance $(\varphi/r) =$ .3ƐƐ73ꞁ98⋮	 {1 2 3 4 5 6 7 8 / ρ ϱ Ɛ ∂ ɣ ꞁ Շ 9} $\varphi =$ ρ


Divisors (leverage)
Balance Study
Totatives (resistance)
2·3²
Balance
(σ0/φ) = 1
Resistance
σ0 = 6
φ = 6
Balance
Resistance
σ0 = 2
2²·5
Balance
Resistance
σ0 = 6
φ = 8
3·7
Balance
Resistance
σ0 = 4


Divisors (leverage)
Balance Study
Totatives (resistance)
Balance
Resistance
σ0 = 4
Balance
Resistance
σ0 = 2
2³·3
Balance
(σ0/φ) = 1
Resistance
σ0 = 8
φ = 8
5²
Balance
(σ0/φ) = .6
Resistance
σ0 = 3

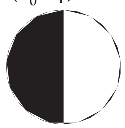
Divisors (leverage)	Balance Study	Totatives (resistance)



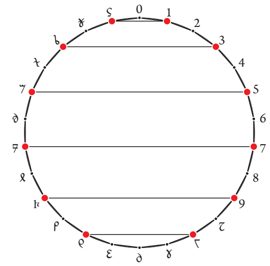
Ɛ

2·∂

Balance
(σ_0/φ) = .∂

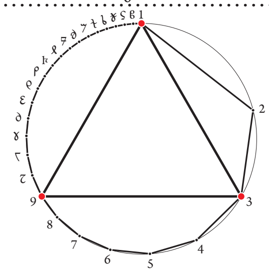


Resistance
(φ/r) = .ʒ⊦8ℓ⋮



{1 2 / Ɛ ∂}
$\sigma_0 = 4$

{1 3 5 7 9 ↊ / Ƨ ♭ 7 ⊦ ⊧ ℘}
φ = Ɣ

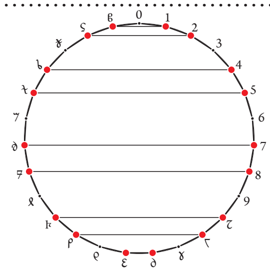


ʒ

3^3

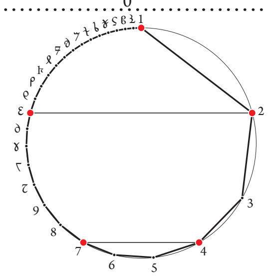
Balance
(σ_0/φ) = .6Ⴘ

Resistance
(φ/r) = .Ⴘ



{1 3 / ʒ 9}
$\sigma_0 = 4$

{1 2 4 5 7 8 ↊ ↋ ∂ / Ɛ Ƨ ♭ ⊹ ∂ 7 ⊦ ρ ε}
φ = ℓ

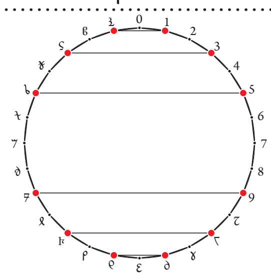


ε

2^2·7

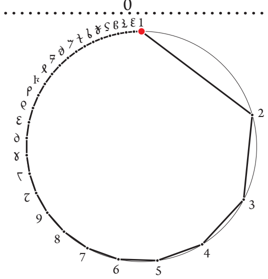
Balance
(σ_0/φ) = .Ꞔ

Resistance
(φ/r) = .ƧƋ⊧⋮



{1 2 4 / ε ε 7}
$\sigma_0 = 6$

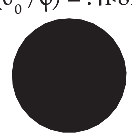
{1 3 5 9 ↊ ∂ / ʒ Ƨ ♭ 7 ⊦ ℘}
φ = Ɣ



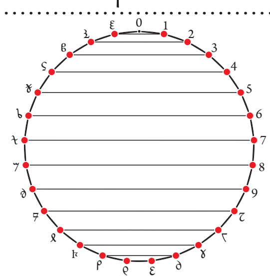
Ɩ

Ɩ

Balance
(σ_0/φ) = .4⊦8⊧⋮



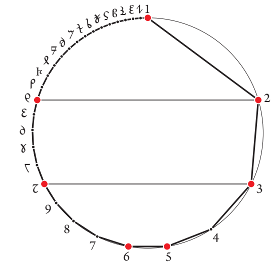
Resistance
(φ/r) = .ℱƃƘƲʒ…



{1 / Ɩ}
$\sigma_0 = 2$

{1 2 3 4 5 6 7 8 9 ↊ Ɣ ∂ ε / ε ʒ Ɛ Ƨ ✗ ♭ ⊹ 7 ∂ 7 ℓ ⊦ ρ ℘}
φ = ε

Divisors (leverage)	Balance Study	Totatives (resistance)

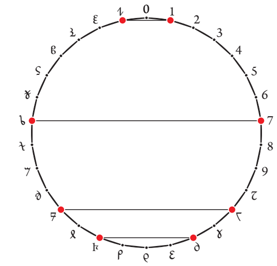


Ꞔ

2·3·5

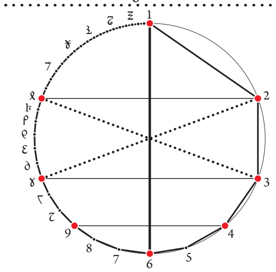
Balance
(σ_0/φ) = .✗

Resistance
(φ/r) = .ρ



{1 2 3 5 / Ꞔ ℘ ↊ 6}
$\sigma_0 = 8$

{1 7 ↊ ∂ / Ɩ ♭ 7 ⊦}
φ = 8

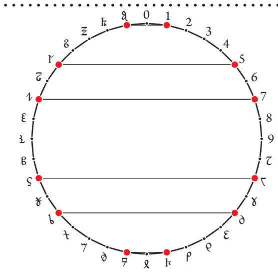


ℓ

$2^2·3^2$

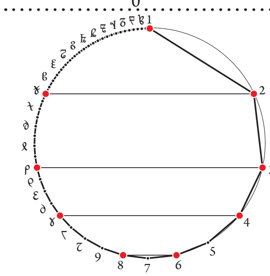
Balance
(σ_0/φ) = .ℓ

Resistance
(φ/r) = .∂



{1 2 3 4 6 / ℓ ℓ Ɣ 9}
$\sigma_0 = 9$

{1 5 7 ↊ ∂ ⊦ / ℌ Ɩ Ɩ Ƨ ♭ 7}
φ = Ɣ

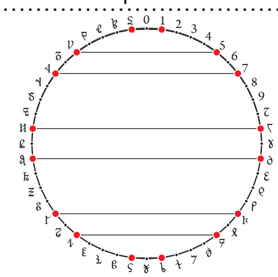


Ɣ

2^4·3

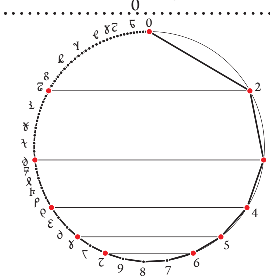
Balance
(σ_0/φ) = .ℋꞔ

Resistance
(φ/r) = .∂



{1 2 3 4 6 / Ɣ Ɣ ρ Ɣ 8}
σ_0 = ↊

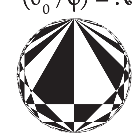
{1 5 7 ↊ ∂ ⊦ 7 ♭ / ℒ Ʋ ⊦ ℋ ℌ Ɩ Ɩ Ƨ}
φ = ρ



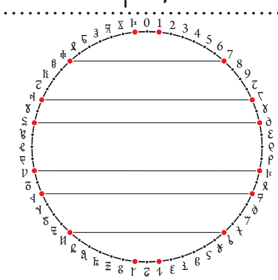
ℊ

2^2·3·5

Balance
(σ_0/φ) = .ℓ



Resistance
(φ/r) = .ρ



{1 2 3 4 5 6 / Ɣ Ꞔ ∂ ℘ Ɣ ↊}
σ_0 = Ɣ

{1 7 ↊ ∂ ⊦ 7 ♭ Ɩ / ⊦ ⊕ ℲℒƲ ⊦ ℋ Ɩ}
φ = ρ

Note

The work embodied in this document consists of amateur observations for incorporation in a larger compendium of such observations. The observations aim to explore alternative number bases, in an attempt to discern which base is the optimum radix for general human computation. Other objectives include the development of methods by which computation becomes feasible in number bases rich in divisors, with a compact and diverse set of such divisors. The observations employ several techniques and tools such as the "argam" or "arqam" numerals and the "Reciprocal Divisor Method" outlined in other portions of the compendium of observations.

The observations are conducted in sexagesimal notation unless otherwise noted, using the argam numerals. Positive integers are normally represented by argam numerals, thus, a number outside the range of the sexagesimal like sixty-one (𐅁) are represented as "single digits". Other observations employ dozenal, base 120, 360, or 2520, and in other bases; these are normally noted. The observations in the compendium took place originally in a physical notebook between late 2006 and mid 2009. The compendium is an ongoing work which will be published at www.vincico.com (Mr. DᵉVlieger's company website). At the time of writing, there is no index to the works at that website.

Decimal-Sexagesimal Argam conversion table:

0	0	𐅀	15	𐅆	30	𐅌	45
1	1	𐅁	16	𐅇	31	𐅍	46
2	2	𐅂	17	𐅈	32	𐅎	47
3	3	𐅃	18	𐅉	33	𐅏	48
4	4	𐅄	19	𐅊	34	𐅐	49
5	5	𐅅	20	𐅋	35	𐅑	50
6	6	𐅆	21	𐅌	36	𐅒	51
7	7	𐅇	22	𐅍	37	𐅓	52
8	8	𐅈	23	𐅎	38	𐅔	53
9	9	𐅉	24	𐅏	39	𐅕	54
𐅊	10	𐅊	25	𐅐	40	𐅖	55
𐅋	11	𐅋	26	𐅑	41	𐅗	56
𐅌	12	𐅌	27	𐅒	42	𐅘	57
𐅍	13	𐅍	28	𐅓	43	𐅙	58
𐅎	14	𐅎	29	𐅔	44	𐅚	59

This work is not intended to be used as an academic resource; please consult a mathematics professional before incorporating it in your own research. It simply serves as a record of an amateur's observations.

This work was produced between third quarter 2008 and 4 March 2010 in Saint Louis, Missouri by Michael Thomas DᵉVlieger, AIA, AIGA. The original source for this work is http://www.vincico.com/arqam/BalanceStudy.pdf. The work was published 4 March 2010. Contact dozenal@vincico.com for questions or to report errata.