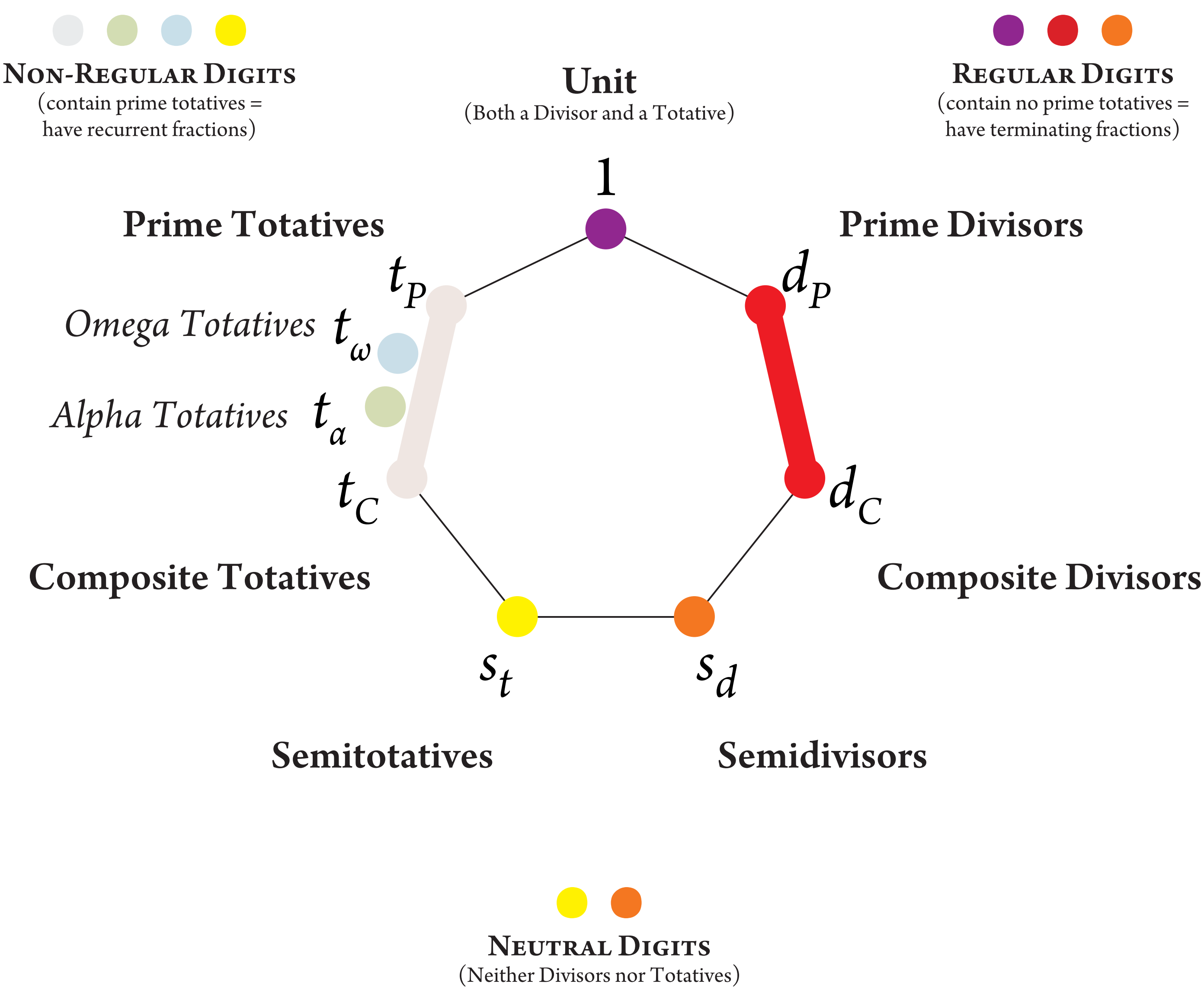
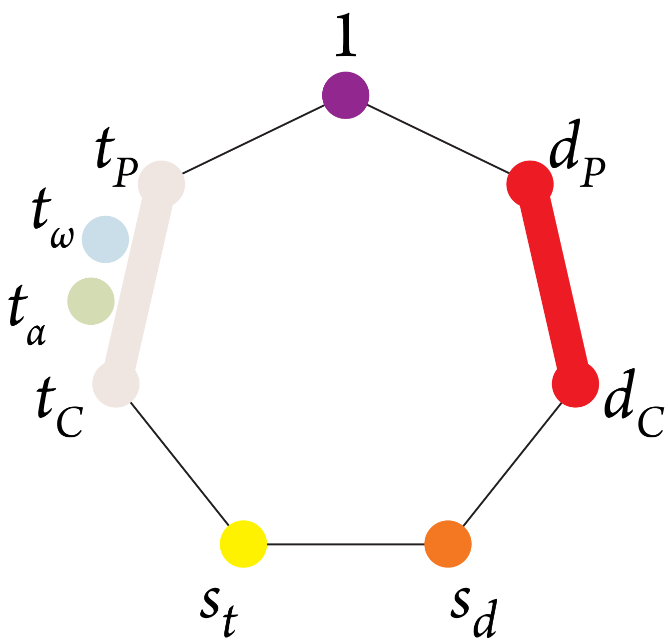
Digit-Base Relationships

A Guide toward Measuring Radix Utility?





Legend

1 **Unit**, $1 \mid r$ and $1 \perp r$

d **Divisor**, primes $d_p \mid r$, and composites $d_c \mid r$, the set of divisors of $r = \mathcal{D}_r$

s_d **Semidivisor**, a composite $s_d \mid kr$, where its divisors $d_s \in \mathcal{D}_r$

s_t **Semitotative**, a composite $s_t \mid kr$, where at least one of its divisors $d_s \notin \mathcal{D}_r$

t **Totative**, primes $t_p \perp r$, and composites $t_c \perp r$, the set of totatives of $r = \mathcal{T}_r$

t_α **α-Totative**, $t_\alpha \in \mathcal{T}_r$ and $t_\alpha \mid (r+1)$

t_ω **ω-Totative**, $t_\omega \in \mathcal{T}_r$ and $t_\omega \mid (r-1)$

$t_{\alpha\omega}$ α/ω-Totative, $t_{\alpha\omega} \in \mathcal{T}_r$, $t_{\alpha\omega} \mid (r+1)$, and $t_{\alpha\omega} \mid (r-1)$

Decimal
(Base 10)

Divisors

0	1	2	3	4	5	6	7	8	9

Totatives

0	1	2	3	4	5	6	7	8	9

Divisors ∪ Totatives

0	1	2	3	4	5	6	7	8	9

Neutral Digits

0	1	2	3	4	5	6	7	8	9
				2^2		$2 \cdot 3$		2^3	

ω-Totatives

0	1	2	3	4	5	6	7	8	9
									ω

Decimal "Digit Map"

0	1	2	3	4	5	6	7	8	9

Dozenal

(Base 12)

Divisors

0	1	2	3	4	5	6	7	8	9	χ	Ɛ

Totatives

0	1	2	3	4	5	6	7	8	9	χ	Ɛ

Divisors ∪ Totatives

0	1	2	3	4	5	6	7	8	9	χ	Ɛ

Neutral Digits

0	1	2	3	4	5	6	7	8	9	χ	Ɛ
								2^3	3^2	$2 \cdot 5$	

ω-Totatives

0	1	2	3	4	5	6	7	8	9	χ	Ɛ
											ω

Dozenal "Digit Map"

0	1	2	3	4	5	6	7	8	9	χ	Ɛ

Decimal
(Base 10)

0	1	2	3	4	5	6	7	8	9

Dozenal
(Base 12)

0	1	2	3	4	5	6	7	8	9	χ	Ɛ

Hexadecimal
(Base 16)

0	1	2	3	4	5	6	7	8	9	a	b	c	d	e	f

Digit Maps for 2 < *r* < 18;

2	0	1																		
3	0	1	2																	
4	0	1	2	3																
5	0	1	2	3	4															
6	0	1	2	3	4	5														
7	0	1	2	3	4	5	6													
8	0	1	2	3	4	5	6	7												
9	0	1	2	3	4	5	6	7	8											
χ	0	1	2	3	4	5	6	7	8	9	χ									
Ɛ	0	1	2	3	4	5	6	7	8	9	Շ	Ɛ								
10	0	1	2	3	4	5	6	7	8	9	Շ	ɾ	10							
11	0	1	2	3	4	5	6	7	8	9	Շ	ɾ	ɤ	11						
12	0	1	2	3	4	5	6	7	8	9	Շ	ɾ	ɤ	∂	12					
13	0	1	2	3	4	5	6	7	8	9	Շ	ɾ	ɤ	∂	ɛ	13				
14	0	1	2	3	4	5	6	7	8	9	Շ	ɾ	ɤ	∂	ɛ	ϱ	14			
15	0	1	2	3	4	5	6	7	8	9	Շ	ɾ	ɤ	∂	ɛ	ϱ	ρ	15		
16	0	1	2	3	4	5	6	7	8	9	Շ	ɾ	ɤ	∂	ɛ	ϱ	ρ	ꝁ	16	
17	0	1	2	3	4	5	6	7	8	9	Շ	ɾ	ɤ	∂	ɛ	ϱ	ρ	ꝁ	ꝉ	17
18	0	1	2	3	4	5	6	7	8	9	Շ	ɾ	ɤ	∂	ɛ	ϱ	ρ	ꝁ	ꝉ	ꟻ

See Table A1 for an expanded view. Visit http://www.vincico.com/arqam/DigitBaseRelationship.pdf to download a table to base χ0.

Decimal (Base 10)

1	2	3	4	5	6	7	8	9	10
2	4	6	8	10	12	14	16	18	20
3	6	9	12	15	18	21	24	27	30
4	8	12	16	20	24	28	32	36	40
5	10	15	20	25	30	35	40	45	50
6	12	18	24	30	36	42	48	54	60
7	14	21	28	35	42	49	56	63	70
8	16	24	32	40	48	56	64	72	80
9	18	27	36	45	54	63	72	81	90
10	20	30	40	50	60	70	80	90	100

Dozenal (Base 12)

1	2	3	4	5	6	7	8	9	χ	Ɛ	10
2	4	6	8	χ	10	12	14	16	18	1χ	20
3	6	9	10	13	16	19	20	23	26	29	30
4	8	10	14	18	20	24	28	30	34	38	40
5	χ	13	18	21	26	2Ɛ	34	39	42	47	50
6	10	16	20	26	30	36	40	46	50	56	60
7	12	19	24	2Ɛ	36	41	48	53	5χ	65	70
8	14	20	28	34	40	48	54	60	68	74	80
9	16	23	30	39	46	53	60	69	76	83	90
χ	18	26	34	42	50	5χ	68	76	84	92	χ0
Ɛ	1χ	29	38	47	56	65	74	83	92	χ1	Ɛ0
10	20	30	40	50	60	70	80	90	χ0	Ɛ0	100

Hexadecimal (Base 16)

1	2	3	4	5	6	7	8	9	a	b	c	d	e	f	10
2	4	6	8	a	c	e	10	12	14	16	18	1a	1c	1e	20
3	6	9	c	f	12	15	18	1b	1e	21	24	27	2a	2d	30
4	8	c	10	14	18	1c	20	24	28	2c	30	34	38	3c	40
5	a	f	14	19	1e	23	28	2d	32	37	3c	41	46	4b	50
6	c	12	18	1e	24	2a	30	36	3c	42	48	4e	54	5a	60
7	e	15	1c	23	2a	31	38	3f	46	4d	54	5b	62	69	70
8	10	18	20	28	30	38	40	48	50	58	60	68	70	78	80
9	12	1b	24	2d	36	3f	48	51	5a	63	6c	75	7e	87	90
a	14	1e	28	32	3c	46	50	5a	64	6e	78	82	8c	96	a0
b	16	21	2c	37	42	4d	58	63	6e	79	84	8f	9a	a5	b0
c	18	24	30	3c	48	54	60	6c	78	84	90	9c	a8	b4	c0
d	1a	27	34	41	4e	5b	68	75	82	8f	9c	a9	b6	c3	d0
e	1c	2a	38	46	54	62	70	7e	8c	9a	a8	b6	c4	d2	e0
f	1e	2d	3c	4b	5a	69	78	87	96	a5	b4	c3	d2	e1	f0
10	20	30	40	50	60	70	80	90	a0	b0	c0	d0	e0	f0	100

Senary
(Base 6)

1	2	3	4	5	10
2	4	10	12	14	20
3	10	13	20	23	30
4	12	20	24	32	40
5	14	23	32	41	50
10	20	30	40	50	100

Septenary
(Base 7)

1	2	3	4	5	6	10
2	4	6	11	13	15	20
3	6	12	15	21	24	30
4	11	15	22	26	33	40
5	13	21	26	34	42	50
6	15	24	33	42	51	60
10	20	30	40	50	60	100

Octal

(Base 8)

1	2	3	4	5	6	7	10
2	4	6	10	12	14	16	20
3	6	11	14	17	22	25	30
4	10	14	20	24	30	34	40
5	12	17	24	31	36	43	50
6	14	22	30	36	44	52	60
7	16	25	34	43	52	61	70
10	20	30	40	50	60	70	100

Nonary
(Base 9)

1	2	3	4	5	6	7	8	10
2	4	6	8	11	13	15	17	20
3	6	10	13	16	20	23	26	30
4	8	13	17	22	26	31	35	40
5	11	16	22	27	33	38	44	50
6	13	20	26	33	40	46	53	60
7	15	23	31	38	46	54	62	70
8	17	26	35	44	53	62	71	80
10	20	30	40	50	60	70	80	100

Decimal

(Base 10)

1	2	3	4	5	6	7	8	9	10
2	4	6	8	10	12	14	16	18	20
3	6	9	12	15	18	21	24	27	30
4	8	12	16	20	24	28	32	36	40
5	10	15	20	25	30	35	40	45	50
6	12	18	24	30	36	42	48	54	60
7	14	21	28	35	42	49	56	63	70
8	16	24	32	40	48	56	64	72	80
9	18	27	36	45	54	63	72	81	90
10	20	30	40	50	60	70	80	90	100

Undecimal
(Base 11)

1	2	3	4	5	6	7	8	9	χ	10
2	4	6	8	χ	11	13	15	17	19	20
3	6	9	11	14	17	1χ	22	25	28	30
4	8	11	15	19	22	26	2χ	33	37	40
5	χ	14	19	23	28	32	37	41	46	50
6	11	17	22	28	33	39	44	4χ	55	60
7	13	1χ	26	32	39	45	51	58	64	70
8	15	22	2χ	37	44	51	59	66	73	80
9	17	25	33	41	4χ	58	66	74	82	90
χ	19	28	37	46	55	64	73	82	91	χ0
10	20	30	40	50	60	70	80	90	χ0	100

Dozenal

(Base 12)

1	2	3	4	5	6	7	8	9	χ	Ɛ	10
2	4	6	8	χ	10	12	14	16	18	1χ	20
3	6	9	10	13	16	19	20	23	26	29	30
4	8	10	14	18	20	24	28	30	34	38	40
5	χ	13	18	21	26	2Ɛ	34	39	42	47	50
6	10	16	20	26	30	36	40	46	50	56	60
7	12	19	24	2Ɛ	36	41	48	53	5χ	65	70
8	14	20	28	34	40	48	54	60	68	74	80
9	16	23	30	39	46	53	60	69	76	83	90
χ	18	26	34	42	50	5χ	68	76	84	92	χ0
Ɛ	1χ	29	38	47	56	65	74	83	92	χ1	Ɛ0
10	20	30	40	50	60	70	80	90	χ0	Ɛ0	100

Tridecimal

(Base 13)

1	2	3	4	5	6	7	8	9	↊	Ꞁ	ɣ	10
2	4	6	8	↊	ɣ	11	13	15	17	19	1Ꞁ	20
3	6	9	ɣ	12	15	18	1Ꞁ	21	24	27	2↊	30
4	8	ɣ	13	17	1Ꞁ	22	26	2↊	31	35	39	40
5	↊	12	17	1ɣ	24	29	31	36	3Ꞁ	43	48	50
6	ɣ	15	1Ꞁ	24	2↊	33	39	42	48	51	57	60
7	11	18	22	29	33	3↊	44	4Ꞁ	55	5ɣ	66	70
8	13	1Ꞁ	26	31	39	44	4ɣ	57	62	6↊	75	80
9	15	21	2↊	36	42	4Ꞁ	57	63	6ɣ	78	84	90
↊	17	24	31	3Ꞁ	48	55	62	6ɣ	79	86	93	↊0
Ꞁ	19	27	35	43	51	5ɣ	6↊	78	86	94	↊2	Ꞁ0
ɣ	1Ꞁ	2↊	39	48	57	66	75	84	93	↊2	Ꞁ1	ɣ0
10	20	30	40	50	60	70	80	90	↊0	Ꞁ0	ɣ0	100

Tetradecimal

(Base 14)

1	2	3	4	5	6	7	8	9	Շ	ᒣ	ɤ	∂	10
2	4	6	8	Շ	ɤ	10	12	14	16	18	1Շ	1ɤ	20
3	6	9	ɤ	11	14	17	1Շ	1∂	22	25	28	2ᒣ	30
4	8	ɤ	12	16	1Շ	20	24	28	2ɤ	32	36	3Շ	40
5	Շ	11	16	1ᒣ	22	27	2ɤ	33	38	3∂	44	49	50
6	ɤ	14	1Շ	22	28	30	36	3ɤ	44	4Շ	52	58	60
7	10	17	20	27	30	37	40	47	50	57	60	67	70
8	12	1Շ	24	2ɤ	36	40	48	52	5Շ	64	6ɤ	76	80
9	14	1∂	28	33	3ɤ	47	52	5ᒣ	66	71	7Շ	85	90
Շ	16	22	2ɤ	38	44	50	5Շ	66	72	7ɤ	88	94	Շ0
ᒣ	18	25	32	3∂	4Շ	57	64	71	7ɤ	89	96	Շ3	ᒣ0
ɤ	1Շ	28	36	44	52	60	6ɤ	7Շ	88	96	Շ4	ᒣ2	ɤ0
∂	1ɤ	2ᒣ	3Շ	49	58	67	76	85	94	Շ3	ᒣ2	ɤ1	∂0
10	20	30	40	50	60	70	80	90	Շ0	ᒣ0	ɤ0	∂0	100

Pentadecimal

(Base 15)

1	2	3	4	5	6	7	8	9	Շ	Γ	ɤ	∂	Ɛ	10
2	4	6	8	Շ	ɤ	Ɛ	11	13	15	17	19	1Γ	1∂	20
3	6	9	ɤ	10	13	16	19	1ɤ	20	23	26	29	2ɤ	30
4	8	ɤ	11	15	19	1Ɛ	22	26	2Շ	2Ɛ	33	37	3Γ	40
5	Շ	10	15	1Շ	20	25	2Շ	30	35	3Շ	40	45	4Շ	50
6	ɤ	13	19	20	26	2ɤ	33	39	40	46	4ɤ	53	59	60
7	Ɛ	16	1Ɛ	25	2ɤ	34	3Γ	43	4Շ	52	59	61	68	70
8	11	19	22	2Շ	33	3Γ	44	4ɤ	55	5∂	66	6Ɛ	77	80
9	13	1ɤ	26	30	39	43	4ɤ	56	60	69	73	7ɤ	86	90
Շ	15	20	2Շ	35	40	4Շ	55	60	6Շ	75	80	8Շ	95	Շ0
Γ	17	23	2Ɛ	3Շ	46	52	5∂	69	75	81	8ɤ	98	Շ4	Γ0
ɤ	19	26	33	40	4ɤ	59	66	73	80	8ɤ	99	Շ6	Γ3	ɤ0
∂	1Γ	29	37	45	53	61	6Ɛ	7ɤ	8Շ	98	Շ6	Γ4	ɤ2	∂0
Ɛ	1∂	2ɤ	4Γ	4Շ	59	68	77	86	95	Շ4	Γ3	ɤ2	∂1	Ɛ0
10	20	30	40	50	60	70	80	90	Շ0	Γ0	ɤ0	∂0	Ɛ0	100

Hexadecimal

(Base 16)

1	2	3	4	5	6	7	8	9	a	b	c	d	e	f	10
2	4	6	8	a	c	e	10	12	14	16	18	1a	1c	1e	20
3	6	9	c	f	12	15	18	1b	1e	21	24	27	2a	2d	30
4	8	c	10	14	18	1c	20	24	28	2c	30	34	38	3c	40
5	a	f	14	19	1e	23	28	2d	32	37	3c	41	46	4b	50
6	c	12	18	1e	24	2a	30	36	3c	42	48	4e	54	5a	60
7	e	15	1c	23	2a	31	38	3f	46	4d	54	5b	62	69	70
8	10	18	20	28	30	38	40	48	50	58	60	68	70	78	80
9	12	1b	24	2d	36	3f	48	51	5a	63	6c	75	7e	87	90
a	14	1e	28	32	3c	46	50	5a	64	6e	78	82	8c	96	a0
b	16	21	2c	37	42	4d	58	63	6e	79	84	8f	9a	a5	b0
c	18	24	30	3c	48	54	60	6c	78	84	90	9c	a8	b4	c0
d	1a	27	34	41	4e	5b	68	75	82	8f	9c	a9	b6	c3	d0
e	1c	2a	38	46	54	62	70	7e	8c	9a	a8	b6	c4	d2	e0
f	1e	2d	3c	4b	5a	69	78	87	96	a5	b4	c3	d2	e1	f0
10	20	30	40	50	60	70	80	90	a0	b0	c0	d0	e0	f0	100

Digits in the Multiplication Tables

Divisors:

2	4	6	8	10	12	14	16	18	20

5	10	15	20	25	30	35	40	45	50

Decimal

2	4	6	8	χ	10	12	14	16	18	1χ	20
3	6	9	10	13	16	19	20	23	26	29	30
4	8	10	14	18	20	24	28	30	34	38	40

6	10	16	20	26	30	36	40	46	50	56	60

Dozenal

Semidivisors:

4	8	12	16	20	24	28	32	36	40

8	16	24	32	40	48	56	64	72	80

Decimal

8	14	20	28	34	40	48	54	60	68	74	80
9	16	23	30	39	46	53	60	69	76	83	90

Dozenal

Semitotatives:

6	12	18	24	30	36	42	48	54	60

Decimal

χ	18	26	34	42	50	5χ	68	76	84	92	χ0

Dozenal

Totatives:

7	14	21	28	35	42	49	56	63	70

Decimal

7	12	19	24	2Ɛ	36	41	48	53	5χ	65	70

Dozenal

ω-Totatives:

3	6	9	12	15	18	21	24	27	30

9	18	27	36	45	54	63	72	81	90

Decimal

Ɛ	1χ	29	38	47	56	65	74	83	92	χ1	Ɛ0

Dozenal

3	6	9	c	f	12	15	18	1b	1e	21	24	27	2a	2d	30
5	a	f	14	19	1e	23	28	2d	32	37	3c	41	46	4b	50
f	1e	2d	3c	4b	5a	69	78	87	96	a5	b4	c3	d2	e1	f0

Hexadecimal

α-Totatives:

4	8	ɤ	11	15	19	1∂	22	26	2Ꞇ	2Ɛ	33	37	3ᒣ	40
8	11	19	22	2Ꞇ	33	3ᒣ	44	4ɤ	55	5∂	66	6Ɛ	77	80

Pentadecimal

α/ω-Totatives:

2	4	6	8	Ꞇ	ɤ	Ɛ	11	13	15	17	19	1ᒣ	1∂	20

Pentadecimal

Digital Fractions

Divisors:

$$^1/_5 = .2_{\text{ten}}$$

$$^1/_3 = ;4_{\text{twelve}}$$

$$^1/_4 = .4_{\text{sixteen}}$$

Semidivisors:

$$^1/_8 = .125_{\text{ten}}$$

$$^1/_8 = ;16_{\text{twelve}}$$

$$^1/_4 = .13_{\text{six}}$$

Totatives:

$$^1/_7 = .142857\ldots_{\text{ten}}$$

$$^1/_7 = ;186\chi35\ldots_{\text{twelve}}$$

Semitotatives:

$$^1/_6 = .16\ldots_{\text{ten}}$$

$$^1/_\chi = ;12497\ldots_{\text{twelve}}$$

$$^1/_C = .15\ldots_{\text{sixteen}}$$

ω-Totatives:

$$^1/_3 = .3\ldots_{\text{ten}}$$

$$^1/_{\mathcal{E}} = ;1\ldots_{\text{twelve}}$$

$$^1/_5 = .3\ldots_{\text{sixteen}}$$

α-Totatives:

$$^1/_3 = .25\ldots_{\text{eight}}$$

$$^1/_5 = ;17\ldots_{\text{nine}}$$

$$^1/_3 = .49\ldots_{\text{fourteen}}$$

α/ω-Totatives:

$$^1/_2 = .55\ldots_{\text{eleven}}$$

Intuitive Divisibility Rules

Rule for Regular Digits and Numbers:

a. Examine the least significant places of an integer.
b. Compare them to a table of multiples of the number

Divisors:

Consider only one least significant place:

0	2	4	6	8

18,672 →

is this place value among the digits which are decimal multiples of 2?

Semidivisors:

Consider multiple least significant places:

15,384 →

00	04	08	12	16
20	24	28	32	36
40	44	48	52	56
60	64	68	72	76
80	84	88	92	96

is this place value group among the 25 two-figure place value groups which are decimal multiples of 4?

Note: "Regular Divisibility Rules" are NOT always practical: consider the decimal divisibility tests for the semidivisor 8, which requires three places be taken into account and 125 possible values to match.

Intuitive Divisibility Rules

Rule for Factors of Radix Neighbors:

1. Separate the places in a number to be tested.
2. Sum the places in one of two ways.
3. If the sum of places is divisible by the subject, the number to be tested is also divisible by the subject.

ω-Totatives:

The "Digit Sum" or "Omega" Rule

DECIMAL TEST FOR DIVISIBILITY BY 3

672 → 6+7+2 → 15

0	3	6	9	12	**15**	18	

is this sum itself divisible by 3?

α-Totatives:

The "Alternating Digit" or "Alpha" Rule

NONARY (base 9) TEST FOR DIVISIBILITY BY 5

4252 → 4+5 → 10; 2+2 → -4 → 5

0	**5**	11	16	

is this sum itself divisible by 5?

α/ω-Totatives:

Either the "Digit Sum"
or the "Alternating Digit" Rule

Note: "Neighbor Related Divisibility Rules" may employ the digital root to arrive at single-place-value results.

Intuitive Divisibility Rules

Compound Rules:

1. If intuitive rules are available for a mutually-coprime set of factors of a composite digit or number, a compound intuitube divisibility test exists for that digit or number

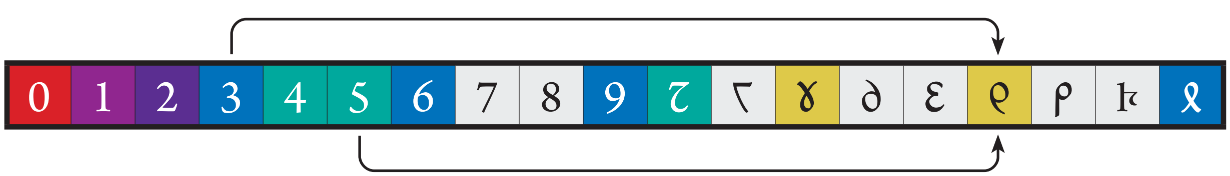
Semitotatives:

Decimal possesses a compound intuitive divisibility test for 6.
Six inherits components of this compound test
from the decimal divisor 2 and the decimal omega totative 3.

6 | 672 *since 672 is even and divisible by three.*

Totatives:

Base nineteen possesses a compound intuitive divisibility test for fifteen.
Digit-fifteen inherits components of this compound test
from the nonodecimal alpha totative 3 and the omega totative 5.



Note: Compound intuitive divisibility rules for totatives seem rare for bases less than ten dozen.

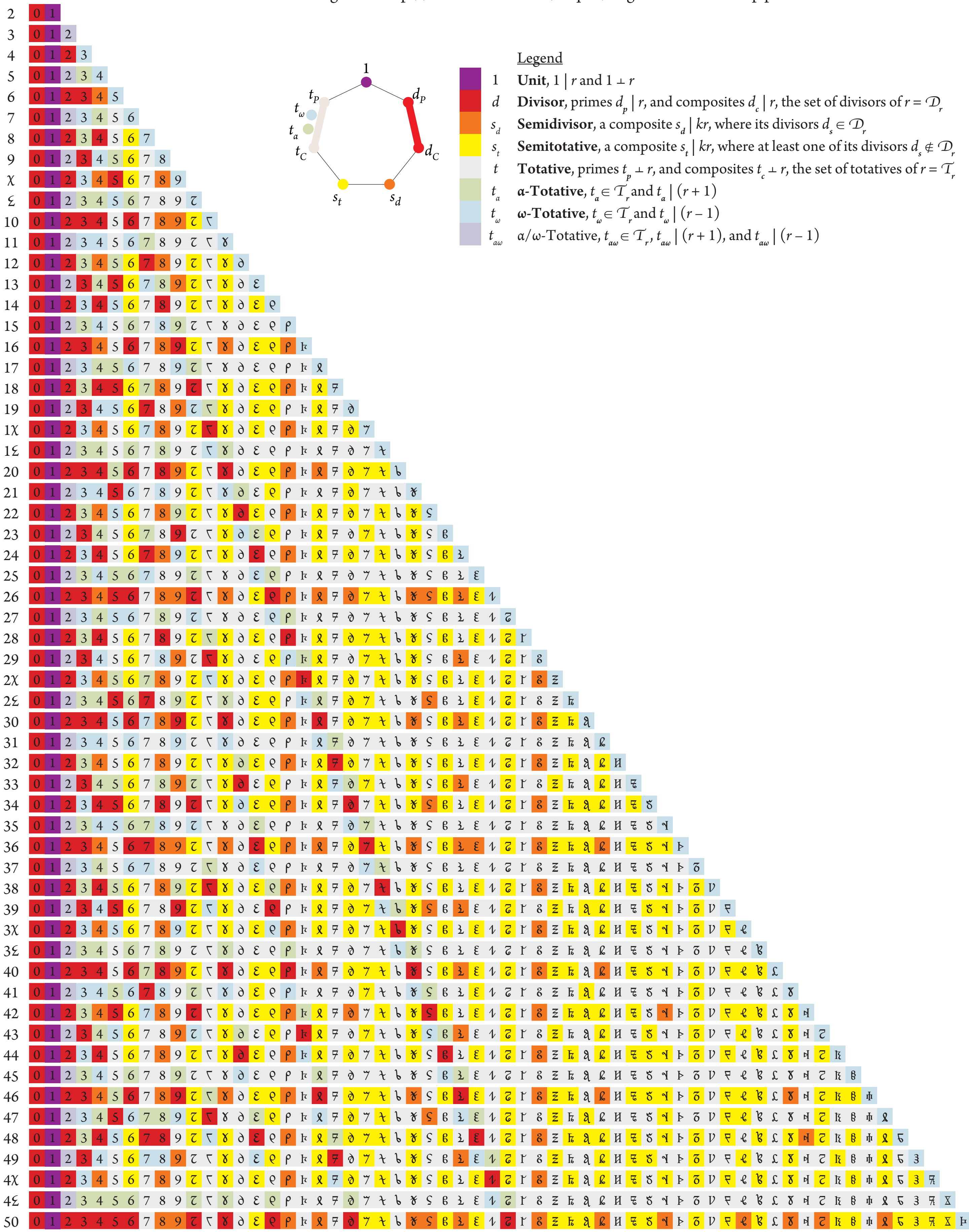
Divisibility Rule Maps for $2 < r < 18$;

											χ	Ƹ	10	11	12	13	14	15	16	17
2	0	1																		
3	0	1	2																	
4	0	1	2	3																
5	0	1	2	3	4															
6	0	1	2	3	4	5														
7	0	1	2	3	4	5	6													
8	0	1	2	3	4	5	6	7												
9	0	1	2	3	4	5	6	7	8											
χ	0	1	2	3	4	5	6	7	8	9										
Ƹ	0	1	2	3	4	5	6	7	8	9	ᘔ									
10	0	1	2	3	4	5	6	7	8	9	ᘔ	ʕ								
11	0	1	2	3	4	5	6	7	8	9	ᘔ	ʕ	ɣ							
12	0	1	2	3	4	5	6	7	8	9	ᘔ	ʕ	ɣ	∂						
13	0	1	2	3	4	5	6	7	8	9	ᘔ	ʕ	ɣ	∂	Ɛ					
14	0	1	2	3	4	5	6	7	8	9	ᘔ	ʕ	ɣ	∂	Ɛ	ϱ				
15	0	1	2	3	4	5	6	7	8	9	ᘔ	ʕ	ɣ	∂	Ɛ	ϱ	ρ			
16	0	1	2	3	4	5	6	7	8	9	ᘔ	ʕ	ɣ	∂	Ɛ	ϱ	ρ	ꝁ		
17	0	1	2	3	4	5	6	7	8	9	ᘔ	ʕ	ɣ	∂	Ɛ	ϱ	ρ	ꝁ	ℓ	
18	0	1	2	3	4	5	6	7	8	9	ᘔ	ʕ	ɣ	∂	Ɛ	ϱ	ρ	ꝁ	ℓ	Ŧ

See Table A11 for an expanded view. Visit http://www.vincico.com/arqam/DigitBaseRelationship.pdf to download an original.

(Table A1)

Relationship of Digits n to Bases r



Legend

1	**Unit**, $1 \mid r$ and $1 \perp r$
d	**Divisor**, primes $d_p \mid r$, and composites $d_c \mid r$, the set of divisors of $r = \mathcal{D}_r$
s_d	**Semidivisor**, a composite $s_d \mid kr$, where its divisors $d_s \in \mathcal{D}_r$
s_t	**Semitotative**, a composite $s_t \mid kr$, where at least one of its divisors $d_s \notin \mathcal{D}_r$
t	**Totative**, primes $t_p \perp r$, and composites $t_c \perp r$, the set of totatives of $r = \mathcal{T}_r$
t_α	**α-Totative**, $t_\alpha \in \mathcal{T}_r$ and $t_\alpha \mid (r+1)$
t_ω	**ω-Totative**, $t_\omega \in \mathcal{T}_r$ and $t_\omega \mid (r-1)$
$t_{\alpha\omega}$	α/ω-Totative, $t_{\alpha\omega} \in \mathcal{T}_r$, $t_{\alpha\omega} \mid (r+1)$, and $t_{\alpha\omega} \mid (r-1)$

Table A2: Digit Spectra for $r \leq$ Sexagesimal

r	PF	Sparkline	R	D	S_d	S_t	$T_n{}^*$	T^*	T
2	2		2	2	·	·	·	·	1
3	3		2	2	·	·	1	·	2
4	2^2		3	3	·	·	1	·	2
5	5		2	2	·	·	3	·	4
6	2 · 3		5	4	1	·	1	·	2
7	7		2	2	·	·	4	1	6
8	2^3		4	4	·	1	2	1	4
9	3^2		3	3	·	1	4	1	6
χ	2 · 5		6	4	2	1	2	1	4
Ɛ	Ɛ		2	2	·	·	4	5	χ
10	2^2 · 3		8	6	2	1	1	2	4
11	11		2	2	·	·	6	5	10
12	2 · 7		6	4	2	3	3	2	6
13	3 · 5		5	4	1	3	5	2	8
14	2^4		5	5	·	4	3	4	8
15	15		2	2	·	·	7	8	14
16	2 · 3^2		χ	6	4	3	1	4	6
17	17		2	2	·	·	8	9	16
18	2^2 · 5		8	6	2	5	3	4	8
19	3 · 7		5	4	1	5	6	5	10
1χ	2 · Ɛ		7	4	3	6	3	6	χ
1Ɛ	1Ɛ		2	2	·	·	8	11	1χ
20	2^3 · 3		Ɛ	8	3	6	2	5	8
21	5^2		3	3	·	3	8	Ɛ	18
22	2 · 11		7	4	3	8	4	7	10
23	3^3		4	4	·	6	6	Ɛ	16
24	2^2 · 7		8	6	2	9	3	8	10
25	25		2	2	·	·	χ	15	24
26	2 · 3 · 5		15	8	9	6	1	6	8
27	27		2	2	·	·	χ	17	26
28	2^5		6	6	·	Ɛ	3	10	14
29	3 · Ɛ		6	4	2	8	6	11	18
2χ	2 · 15		8	4	4	Ɛ	5	χ	14
2Ɛ	5 · 7		5	4	1	7	9	12	20
30	2^2 · 3^2		12	9	5	Ɛ	3	8	10
31	31		2	2	·	·	9	22	30
32	2 · 17		8	4	4	11	3	12	16
33	3 · 11		6	4	2	χ	8	13	20
34	2^3 · 5		Ɛ	8	3	12	3	10	14
35	35		2	2	·	·	10	23	34
36	2 · 3 · 7		17	8	Ɛ	10	1	χ	10
37	37		2	2	·	·	χ	27	36
38	2^2 · Ɛ		9	6	3	14	5	12	18
39	3^2 · 5		8	6	2	12	6	15	20
3χ	2 · 1Ɛ		8	4	4	15	5	14	1χ
3Ɛ	3Ɛ		2	2	·	·	χ	2Ɛ	3χ
40	2^4 · 3		13	χ	5	15	2	11	14
41	7^2		3	3	·	5	10	25	36
42	2 · 5^2		10	6	6	17	4	13	18
43	3 · 15		6	4	2	12	8	1Ɛ	28
44	2^2 · 11		9	6	3	18	3	18	20
45	45		2	2	·	·	χ	35	44
46	2 · 3^2		14	8	8	19	3	12	16
47	5 · Ɛ		5	4	1	Ɛ	10	23	34
48	2^3 · 7		Ɛ	8	3	1χ	5	16	20
49	3 · 17		6	4	2	14	8	23	30
4χ	2 · 25		8	4	4	1Ɛ	3	20	24
4Ɛ	4Ɛ		2	2	·	·	10	39	4χ
50	2^2 · 3 · 5		22	10	12	17	1	12	14

Table A3: Digit Spectra for Primorials

r	PF	Sparkline	R	D	S_d	S_t	T_x^*	T^*	T
2	1		2	2	·	·	·	·	1
6	11		5	4	1	·	1	·	2
26	111		15	8	9	6	1	6	8
156	1111		58	14	44	7Ɛ	3	38	50
1406	11111		182	28	156	105Ɛ	1	33χ	340
15466	111111			54			3		3400
207526	1111111			χ8					45400

Table A4: Digit Spectra for Superior Highly Composite Numbers

r	PF	Sparkline	R	D	S_d	S_t	T_x^*	T^*	T
2	1		2	2	·	·	·	·	1
6	11		5	4	1	·	1	·	2
10	12		8	6	2	1	1	2	4
50	112		22	10	12	17	1	14	14
χ0	113		30	14	18	57	4	23	28
260	123		4χ	20	2χ	153	2	79	80
1560	1123		177	40	χ7	1036	3	3Ɛ8	400
2Ɛ00	1124		23χ	50	1χχ	20Ɛ7	2	7Ɛ9	800
28100	11124			χ0			1	67Ɛχ	6800
2χ9100	111124			180			6	344Ɛ5	34500

Table A5: Digit Spectra for Highly Composite Numbers and Peers

r	PF	Sparkline	R	D	S_d	S_t	T_x^*	T^*	T
2	1		2	2	·	·	·	·	1
3	10		2	2	·	·	1	·	2
4	2		3	3	·	·	1	·	2
6	11		5	4	1	·	1	·	2
8	3		4	4	·	1	2	1	4
χ	101		6	4	2	1	2	1	4
10	12		8	6	2	1	1	2	4
16	21		χ	6	4	3	1	4	6
18	102		8	6	2	5	3	4	8
20	13		Ɛ	8	3	6	2	5	8
26	111		15	8	9	6	1	6	8
30	22		12	9	5	11	3	8	10
40	13		13	χ	5	15	2	11	14
50	112		22	10	12	17	1	12	14
60	23		16	10	6	26	1	1χ	20
70	1012		23	10	13	29	3	18	20
76	121		19	10	9	39	3	18	20
80	15		18	10	8	38	3	24	28
90	32		18	10	8	44	1	2χ	30
χ0	113		30	14	18	57	4	23	28
120	1013		31	14	19	6Ɛ	2	39	40
130	122		38	16	22	74	1	3χ	40
180	114		33	18	25	Ɛ5	1	52	54
240	1014		42	18	26	13χ	3	78	80
260	123		4χ	20	2χ	153	2	79	80
2Ɛ0	1112		Ɛ9	20	99	134	1	7χ	80
340	115		56	20	36	1ƐƐ	3	χ4	χ8
360	1023		50	20	30	211	3	Ɛ8	100
420	213		5Ɛ	20	3Ɛ	26χ	1	112	114
446	1121		97	20	77	270	3	Ɛ8	100
470	10112		8χ	20	6χ	28Ɛ	1	112	114
480	1015		56	20	36	2χ7	3	138	140
500	124		63	26	39	31χ	3	138	140
5χ0	1113		Ɛ0	28	84	371	2	139	140
760	133		74	28	48	535	5	1Ɛ6	200
890	1122		96	30	66	5Ɛ7	3	1Ɛ8	200
χ00	125		81	30	51	500	3	278	280
Ɛ80	1114		Ɛ0	34	78	811	4	277	280
1300	134		93	34	5Ɛ	χ89	3	3Ɛ8	400
1560	1123		127	40	χ7	1036	3	3Ɛ8	400

Table A6: Digit Spectra for $r = 2^n$ Where $1 \le n \le 10$

r	PF	Sparkline	R	D	S_d	S_t	T_n^*	T^*	T
2	2		2	2	·	·	·	·	1
4	2^2		3	3	·	·	1	·	2
8	2^3		4	4	·	1	2	1	4
14	2^4		5	5	·	4	3	4	8
28	2^5		6	6	·	Ɛ	3	10	14
54	2^6		7	7	·	22	7	20	28
χ8	2^7		8	8	·	49	3	50	54
194	2^8		9	9	·	χ0	7	χ0	χ8
368	2^9		χ	χ	·	187	9	186	194
714	$2^χ$		Ɛ	Ɛ	·	35χ	Ɛ	358	368
1228	$2^Ɛ$		10	10	·	695	5	70χ	714
2454	2^{10}		11	11	·	1218	21	1206	1228

Table A7: Digit Spectra for $r = 3^n$ Where $1 \le n \le 8$

r	PF	Sparkline	R	D	S_d	S_t	T_n^*	T^*	T
3	3		2	2	·	·	1	·	2
9	3^2		3	3	·	1	4	1	6
23	3^3		4	4	·	6	6	Ɛ	16
69	3^4		5	5	·	1Ɛ	χ	37	46
183	3^5		6	6	·	64	8	108	116
509	3^6		7	7	·	147	18	329	346
1323	3^7		8	8	·	502	6	χ0Ɛ	χ16
3969	3^8		9	9	·	1317	24	2621	2646

Table A8: Digit Spectra for $r = 6^n$ Where $1 \le n \le 6$

r	PF	Sparkline	R	D	S_d	S_t	T_n^*	T^*	T
6	$2 \cdot 3$		5	4	1	·	1	·	2
30	$2^2 \cdot 3^2$		12	9	5	Ɛ	3	8	10
160	$2^3 \cdot 3^3$		22	14	χ	9Ɛ	5	56	60
900	$2^4 \cdot 3^4$		37	21	16	586	7	2Ɛ4	300
4600	$2^5 \cdot 3^5$		54	30	24	2Ɛ69	Ɛ	15Ɛ0	1600
23000	$2^6 \cdot 3^6$		75	41	34	15Ɛ48	19	8Ɛχ2	9000

Table A9: Digit Spectra for $r = Ƭ^n$ Where $1 \le n \le 4$

r	PF	Sparkline	R	D	S_d	S_t	T_n^*	T^*	T
χ	$2 \cdot 5$		6	4	2	1	2	1	4
84	$2^2 \cdot 5^2$		13	9	6	3χ	5	2χ	34
6Ɛ4	$2^3 \cdot 5^3$		25	14	11	3Ɛ8	13	27χ	294
5954	$2^4 \cdot 5^4$		40	21	1Ɛ	3541	11	2382	2394

Table Aχ: Digit Spectra for $r = Ɣ^n$ Where $1 \le n \le 4$

r	PF	Sparkline	R	D	S_d	S_t	T_n^*	T^*	T
50	$2^2 \cdot 3$		8	6	2	1	1	2	4
2600	$2^4 \cdot 3^2$		1Ɛ	13	8	62	5	36	40
χ5000	$2^6 \cdot 3^3$		3χ	24	16	763	9	3Ɛ2	400
4410000	$2^8 \cdot 3^4$		66	39	29	7Ɛ57	15	3Ɛχ6	4000

Table AƐ: Digit Spectra for $r = 10^n$ Where $1 \le n \le 2$

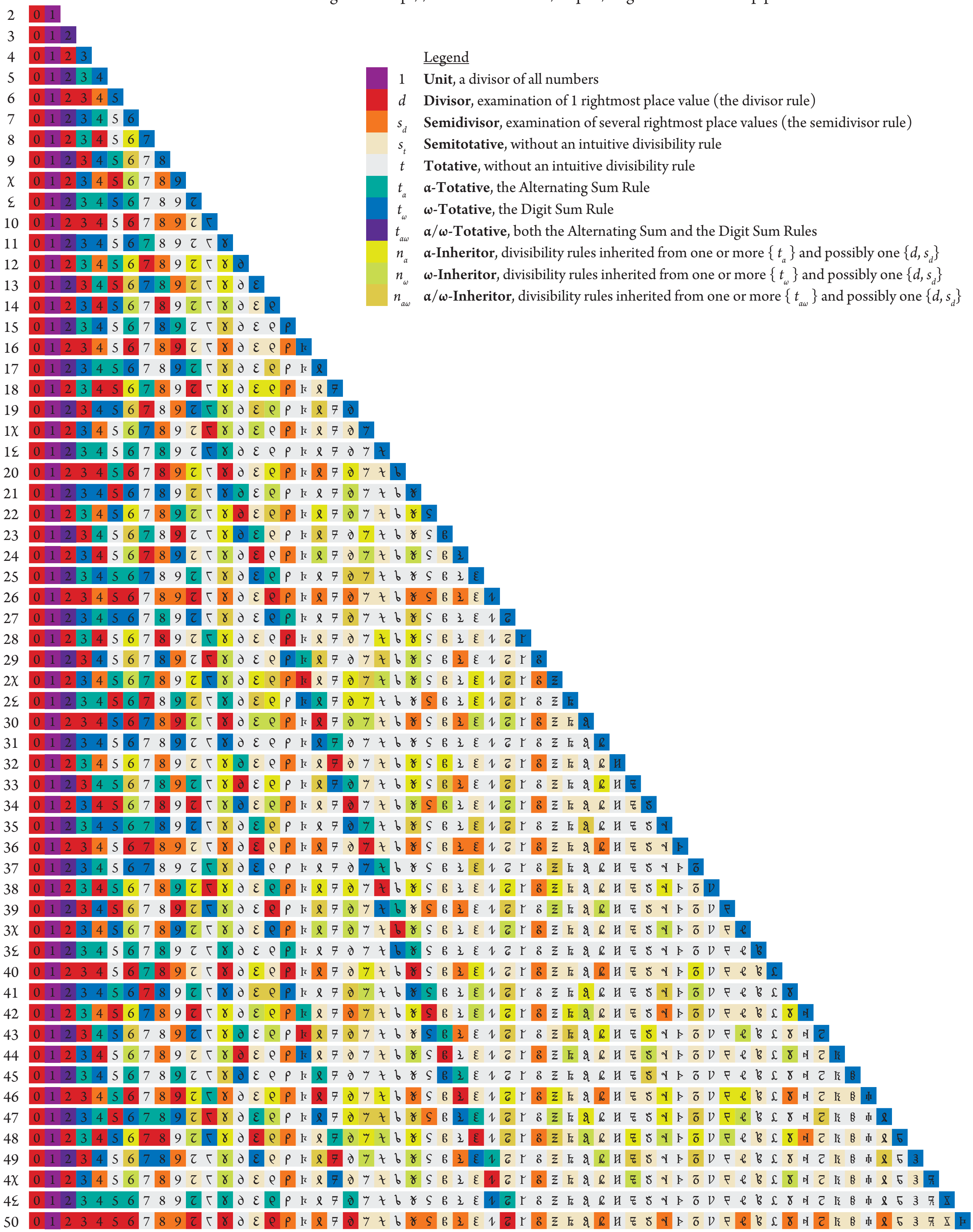
r	PF	Sparkline	R	D	S_d	S_t	T_n^*	T^*	T
50	$2^2 \cdot 3 \cdot 5$		22	10	12	17	1	12	14
2100	$2^4 \cdot 3^2 \cdot 5^2$		χ9	39	100	1554	5	676	680

Table A10: Digit Spectra for Commonly Considered Bases r

r	PF	Sparkline	R	D	S_d	S_t	T_n^*	T^*	T
2	2		2	2	·	·	·	·	1
6	$2 \cdot 3$		5	4	1	·	1	·	2
8	2^3		4	4	·	1	2	1	4
χ	$2 \cdot 5$		6	4	2	1	2	1	4
10	$2^2 \cdot 3$		8	6	2	1	1	2	4
14	2^4		5	5	·	4	3	4	8
18	$2^2 \cdot 5$		8	6	2	5	3	4	8
50	$2^2 \cdot 3 \cdot 5$		22	10	12	17	1	12	14

(Table A11)

Intuitive Divisibility Rules for Digits *n* in Bases *r*



Legend

- 1 — **Unit**, a divisor of all numbers
- d — **Divisor**, examination of 1 rightmost place value (the divisor rule)
- s_d — **Semidivisor**, examination of several rightmost place values (the semidivisor rule)
- s_t — **Semitotative**, without an intuitive divisibility rule
- t — **Totative**, without an intuitive divisibility rule
- t_α — **α-Totative**, the Alternating Sum Rule
- t_ω — **ω-Totative**, the Digit Sum Rule
- $t_{\alpha\omega}$ — **α/ω-Totative**, both the Alternating Sum and the Digit Sum Rules
- n_α — **α-Inheritor**, divisibility rules inherited from one or more $\{t_\alpha\}$ and possibly one $\{d, s_d\}$
- n_ω — **ω-Inheritor**, divisibility rules inherited from one or more $\{t_\omega\}$ and possibly one $\{d, s_d\}$
- $n_{\alpha\omega}$ — **α/ω-Inheritor**, divisibility rules inherited from one or more $\{t_{\alpha\omega}\}$ and possibly one $\{d, s_d\}$